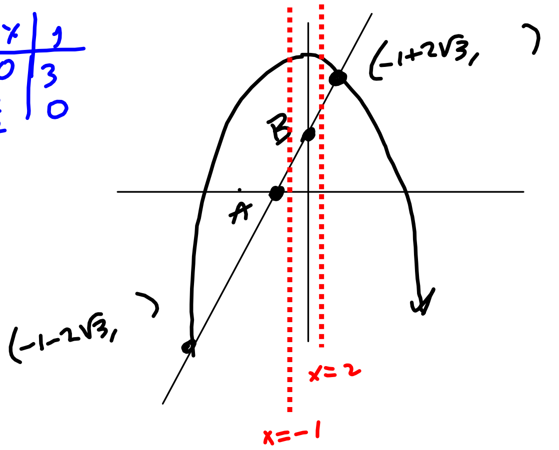
Test 5 Part II Questions

Sketch the region enclosed by the given curves. Decide whether to integrate with respect to x or y. Draw a typical approximating rectangle and label its height and width.

$y = 2x + 3, \quad y = 14 - x^2, \quad x = -1, \quad x = 2$

x	y
0	3
$-\frac{3}{2}$	0



$\sqrt{14} \approx 3.741657387$

Find intersections

$14 - x^2 = 2x + 3$

$-x^2 - 2x + 11 = 0$

$x^2 + 2x - 11 = 0$

$x^2 + 2x + 1^2 = 11 + 1$

$(x+1)^2 = 12$

$x + 1 = \pm\sqrt{12} = \pm 2\sqrt{3}$

$x = -1 \pm 2\sqrt{3} \to 2.464101616$

$\to -4.464101616$

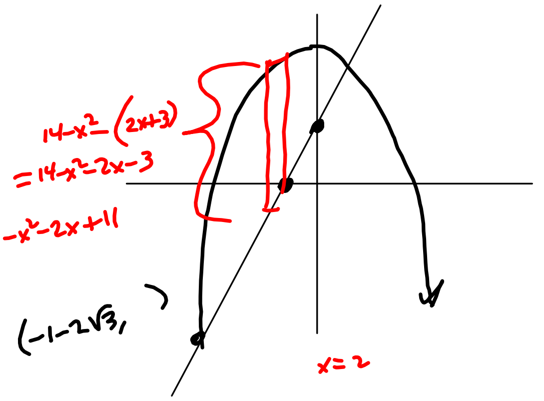
$\int_{-1}^{2} (\text{upper} - \text{lower})$

$= \int_{-1}^{2} (-x^2 - 2x + 11)\, dx$

$14 - x^2 - (2x+3)$

$= 14 - x^2 - 2x - 3$

$= -x^2 - 2x + 11$



$evalf(\text{sqrt}(14))$

3.741657387

$-1 - 2\cdot\text{sqrt}(3)$

$-1 - 2\sqrt{3}$

$evalf(\%)$

-4.464101616

$\int_{-1}^{2} (-x^2 - 2\cdot x + 11)\, dx$

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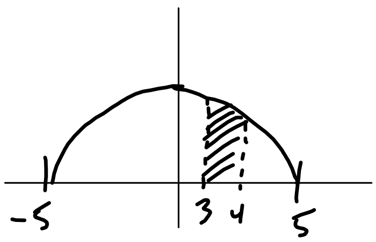
Find the volume V of the solid obtained by rotating the region bounded by the given curves about the specified lin

$$y = 3\sqrt{25 - x^2}, \quad y = 0, \quad x = 3, \quad x = 4, \quad \text{about the } x\text{-axis}$$

$V =$ 114π

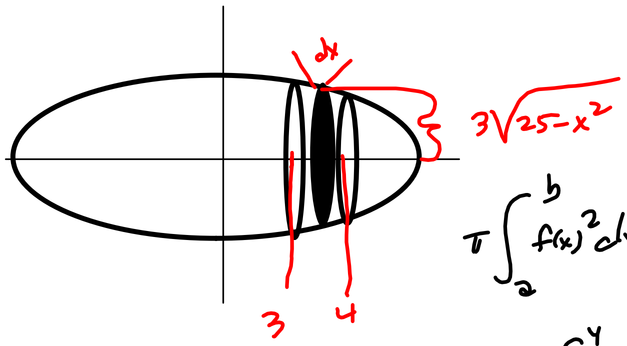
Sketch the region.



$$y^2 = 3^2(25 - x^2)$$
$$y^2 = 225 - 9x^2$$
$$9x^2 + y^2 = 225$$
$$\frac{x^2}{\left(\frac{225}{9}\right)} + \frac{y^2}{225} = 1$$

Find the volume

Sketch the solid



$$\pi \int_a^b f(x)^2\,dx = \pi \int_3^4 \left(3\sqrt{25 - x^2}\right)^2 dx$$

$$= 9\pi \int_3^4 (25 - x^2)\,dx$$

$$= 9\pi \left[25x - \frac{x^3}{3}\right]_3^4$$

$$= 9\pi \left[25(4) - \frac{4^3}{3} - \left(25(3) - \frac{3^3}{3}\right)\right]$$

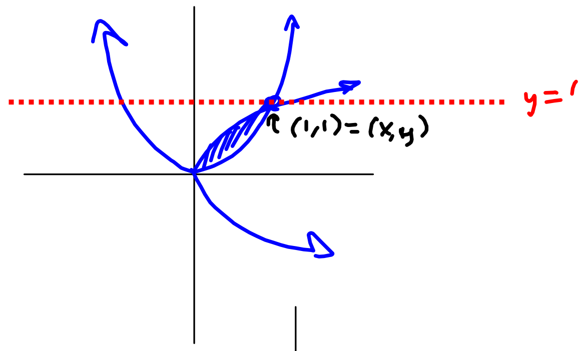
$$= 9\pi \left[25 + \frac{-64 + 27}{3}\right] = 9\pi \left[\frac{75}{3} - \frac{37}{3}\right] = 9\pi \left[\frac{38}{3}\right]$$

$$\begin{array}{r} {}^{7}38 \\ \times\ 9 \\ \hline 342 \end{array} \qquad = \frac{342\pi}{3} = \boxed{114\pi}$$

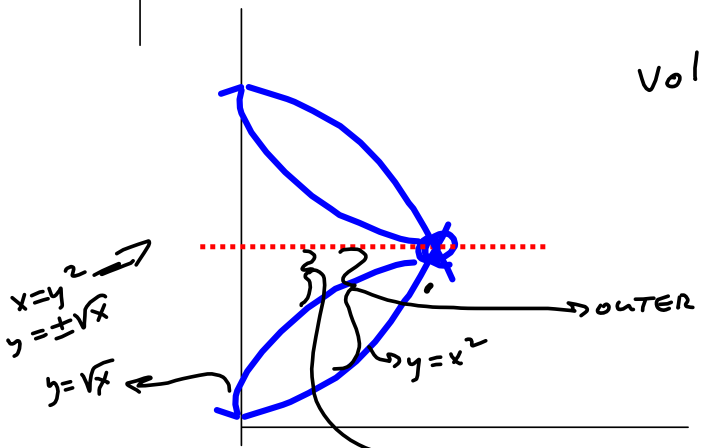
Find the volume V of the solid obtained by rotating the region bounded by the given curves about the specified line.

$y = x^2, \quad x = y^2; \quad \text{about } y = 1$

$V =$ ✖ $\boxed{\frac{11\pi}{30}}$

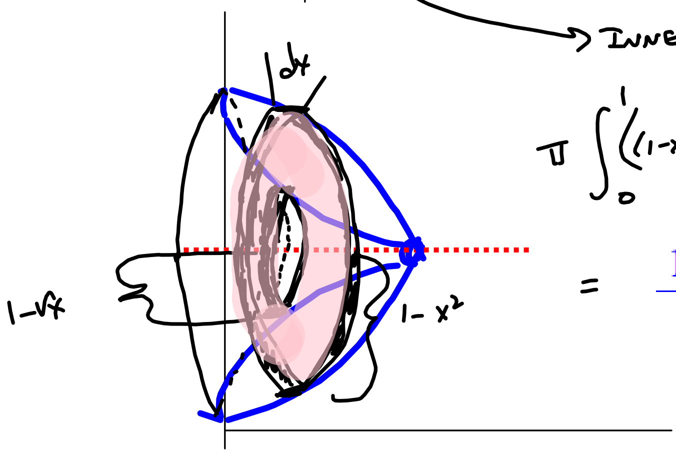


$$\text{Volume} = \pi \int_0^1 \left(\text{OUTER}^2 - \text{INNER}^2\right)$$



OUTER $= 1 - x^2$

INNER $= 1 - \sqrt{x}$



$$\pi \int_0^1 \left((1-x^2)^2 - (1-\sqrt{x})^2\right) dx$$

$$= \frac{11\pi}{30}$$

1-to-1 teaches the contrapositive.

$x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)$ $\quad A \Rightarrow B$

&

$f(x_1) = f(x_2) \Rightarrow x_1 = x_2$ $\quad$ Not B $\Rightarrow$ Not A

Let $f(x) = \frac{4x-1}{2x+3}$. Find f^{-1}. But first prove f is 1-to-1.

Assume $f(x_1) = f(x_2)$. Then

$$\frac{4x_1-1}{2x_1+3} = \frac{4x_2-1}{2x_2+3}$$

$$(4x_1-1)(2x_2+3) = (4x_2-1)(2x_1+3)$$

$$\Rightarrow 8x_1x_2 + 12x_1 - 2x_2 - 3 = 8x_1x_2 + 12x_2 - 2x_1 - 3$$

$$\Rightarrow 14x_1 = 14x_2$$

$$\Rightarrow x_1 = x_2$$

$\Rightarrow$ f is 1-to-1

We get the following.

$$y = f(x) = \frac{4x-1}{2x+3} \Rightarrow$$

$$y(2x+3) = 4x - 1 \Rightarrow$$

$$2xy + 3y = 4x - 1 \Rightarrow$$

$$3y + 1 = 4x - 2xy \Rightarrow$$

$$3y + 1 = (4 - 2y)x \Rightarrow$$

$$x = \frac{3y+1}{4-2y}$$

Interchange x and y: $y = \frac{3x+1}{4-2x}$.

So $f^{-1}(x) = \frac{3x+1}{4-2x}$.

Find f^{-1}:

$$x = \frac{4y-1}{2y+3} = x$$

$$\Rightarrow 4y - 1 = x(2y+3) = 2xy + 3x$$

$$4y - 2xy = 3x + 1$$

$$y(4 - 2x) = 3x + 1$$

$$y = \boxed{\frac{3x+1}{4-2x} = f^{-1}(x)}$$