Derivative of logarithmic functions:

$y = \ln(x) \Rightarrow e^y = x$

$\Rightarrow \frac{d}{dx}[e^y] = e^y y' = 1 \Rightarrow$

$y' = \frac{1}{e^y} = \frac{1}{e^{\ln(x)}} = \frac{1}{x}$!

$\frac{d}{dx}[\ln(x)] = \frac{1}{x}$ and $\int \frac{1}{x}dx = \int \frac{dx}{x} = \ln(x)^* + C$ (so $x>0$ is needed)

See Example 7 in text.

Actually $\boxed{\int \frac{1}{x}dx = \ln|x| + C}$

$u = f(x) \Rightarrow$

$du = f'(x)dx$

$\ln|u| + C = \int \frac{du}{u} = \int \frac{f'(x)}{f(x)}dx = \ln|f(x)| + C$

Recall

$\int \sin(x)dx = -\cos(x) + C$

$\int \cos(x)dx = \sin(x) + C$

$\int \sec^2(x)dx = \tan(x) + C$

$\int \csc^2(x)dx = -\cot(x) + C$

$\int \sec(x)dx = ?$

$\int \csc(x)dx = ?$

$\int \tan(x)dx = ?$

$\int \cot(x)dx = ?$

$\int \tan(x)dx = \int \frac{\sin(x)}{\cos(x)}dx = -\int \frac{-\sin(x)}{\cos(x)}dx = -\int \frac{f'(x)}{f(x)}dx,$

where $f(x) = \cos(x) \Rightarrow$ we have

$$-\ln|f(x)|+C = -\ln|\cos(x)|+C = -1\ln|\cos(x)|+C$$

$$= \ln\left(|\cos(x)|^{-1}\right)+C = \ln|\cos(x)^{-1}|+C$$

$$= \ln\left|\frac{1}{\cos(x)}\right|+C = \boxed{\ln|\sec(x)|+C = \int \tan(x)\,dx}$$

$$\int \sec(x)\,dx = \int \sec(x)\left(\frac{\sec(x)+\tan(x)}{\sec(x)+\tan(x)}\right)dx$$

$$= \int \frac{\sec^2(x)+\sec(x)\tan(x)}{\sec(x)+\tan(x)}\,dx = \int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)|+C$$

Now $f(x) = \sec(x) + \tan(x)$

$\Rightarrow f'(x) = \sec(x)\tan(x) + \sec^2(x)$

$$\boxed{= \ln|\sec(x)+\tan(x)|+C = \int \sec(x)\,dx}$$

Now, you can derive $\int \csc(x)\,dx$

& $\int \cot(x)\,dx$.

Click here for videos and notes for Logarithmic and Exponential Functions in College Algebra

Applications of $\frac{d}{dx}\left[\ln(f(x))\right] = \frac{f'(x)}{f(x)}$

$$\frac{d}{dx}\left[\ln\left(\frac{\sqrt[5]{x^2-5}\,(x^5+2x)^2}{\sin(x)}\right)\right]$$

$$= \frac{d}{dx}\left[\ln\left(\frac{(x^2-5)^{\frac{1}{5}}(x^5+2x)^2}{\sin(x)}\right)\right]$$

$$= \frac{d}{dx}\left[\ln\left((x^2-5)^{\frac{1}{5}}\right) + \ln\left((x^5+2x)^2\right) - \ln(\sin(x))\right]$$

$$= \frac{d}{dx}\left[\frac{1}{5}\ln(x^2-5) + 2\ln(x^5+2x) + \ln(\sin(x))\right]$$

$$= \frac{1}{5}\left(\frac{2x}{x^2-5}\right) + 2\left(\frac{5x^4+2}{x^5+2x}\right) + \frac{\cos(x)}{\sin(x)}$$

$\to \cot(x).$

Final bit of theory for the semester:

Logarithmic Differentiation. Very nice for $\frac{d}{dx}\left[f(x)^{g(x)}\right]$

$\frac{d}{dx}\left[\sin(x)^{\cos(x)}\right] = \frac{d}{dx}[y]$, where

$y = \sin(x)^{\cos(x)} \Longrightarrow$

$\ln(y) = \ln\left(\sin(x)^{\cos(x)}\right) = \cos(x)\ln(\sin(x))$

$\frac{d}{dx}[LHS = RHS]$:

$\frac{y'}{y} = -\sin(x)\ln(\sin(x)) + \cos(x)\cdot\frac{\cos(x)}{\sin(x)} \Longrightarrow$

$y' = \Big($ WHOLE MESS $\Big)\cdot y$

$= \left(-\sin(x)\ln(\sin(x)) + \frac{\cos^2(x)}{\sin(x)}\right)\left(\sin(x)^{\cos(x)}\right) = y'$

$$\frac{d}{dx}\left[\frac{(x^2+5x)^{13}(x^3+11x^2)^7}{(x^5+7)^3(\sin^2(x))}\right] = \frac{d}{dx}[y] \Longrightarrow$$

$$\ln(y) = 13\ln(x^2+5x) + 7\ln(x^3+11x^2) - 3\ln(x^5+7) - 2\ln(\sin(x))$$

$$\Longrightarrow y' = \left(13\left(\frac{2x+5}{x^2+5x}\right) + 7\left(\frac{3x^2+22x}{x^3+11x^2}\right) - 3\left(\frac{5x^4}{x^5+7}\right) - 2\cot(x)\right)\left(\frac{(x^2+5x)^{13}(x^3+11x^2)^7}{(x^5+7)^3(\sin^2(x))}\right)$$

Find y' if $y = \ln(x^2 - \sin(y))$

$$\Longrightarrow y' = \frac{2x - \cos(y)y'}{x^2 - \sin(y)} = \frac{2x}{x^2 - \sin y} - \frac{\cos(y)y'}{x^2 - \sin(y)}$$

$$y' + \frac{\cos(y)y'}{x^2 - \sin(y)} = \frac{2x}{x^2 - \sin(y)} \Longrightarrow$$

$$y'\left(1 + \frac{\cos(y)}{x^2 - \sin(y)}\right) = \frac{2x}{x^2 - \sin(y)}$$

$$y' = \frac{\frac{2x}{x^2 - \sin(y)}}{\frac{x^2 - \sin(y) + \cos(y)}{x^2 - \sin(y)}} = \frac{2x}{\cancel{x^2 - \sin(y)}} \cdot \frac{\cancel{x^2 - \sin(y)}}{x^2 - \sin(y) + \cos(y)} = \boxed{\frac{2x}{x^2 - \sin(y) + \cos(y)} = y'}$$