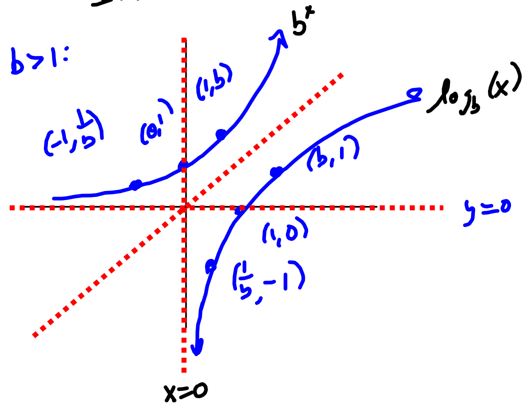
$\log_b(x) = y$ means $x = b^y$

It's the inverse of $y = b^x$



$0 < b < 1$ pic

$\frac{d}{dx}[e^x] = e^x$ $\quad e \approx$ 2.718281828

Recall: $\frac{d}{dx}[b^x] = \lim_{h\to 0} \frac{b^{x+h} - b^x}{h} = \lim_{h\to 0} \frac{b^x b^h - b^x}{h} = \lim_{h \to 0} b^x\left(\frac{b^h - 1}{h}\right)$

If we can evaluate
$\lim_{h\to 0} \frac{b^h - 1}{h}$

$\frac{2^h - 1}{h} \xrightarrow{h\to 0} \ln(2)$

$\frac{3^h - 1}{h} \xrightarrow{h\to 0} \ln(3)$

$\frac{e^h - 1}{h} \xrightarrow{h\to 0} 1$!

The big trick!

e^x & $\log_e(x) = \ln(x)$ are inverses.

$e^{\ln(x)} = x$

$\ln(e^x) = x$

$$\frac{d}{dx}\left[b^x\right] = \left(\ln(b)\right) b^x$$

$$b^x = e^{\ln(b^x)} = e^{x\ln(b)} = e^{(\ln(b))x} = \left(e^{\ln(b)}\right)^x = \left(e^x\right)^{\ln(b)}$$

$$\frac{d}{dx}\left[e^x\right] = e^x$$

~~General~~ Power Rule

$$\cancel{\frac{d}{dx}\left[f(x)^n\right] = n f(x)^{n-1}} f'(x)$$

No. Chain Rule

$$\frac{d}{dx}\left[e^x\right] = e^x \longrightarrow \frac{d}{dx}\left[e^{f(x)}\right] = \left(e^{f(x)}\right)\left(f'(x)\right)$$

$$f(x) = \ln(b)x$$

$$f'(x) = \ln(b)$$

$$\longrightarrow \frac{d}{dx}\left[b^x\right] = \frac{d}{dx}\left[e^{\ln(b)x}\right] = \left[e^{\ln(b)x}\right]\ln(b)$$

$$= \left[e^{\ln(b^x)}\right]\ln(b) = b^x \cdot \ln(b) = \ln(b)\cdot b^x.$$

$$\boxed{\int b^x dx = \frac{1}{\ln b} b^x + C}$$

I never memorized this. I just remembered my properties of exponents/ logarithms. Then I remembered what the derivative/anti-derivative of e^x was.

For derivative of logarithmic function.

$$\log_b(x) = \frac{\ln(x)}{\ln(b)} = \frac{1}{\ln(b)} \ln(x)$$

$\ln(x)$ & e^x are inverses.

What's $\frac{d}{dx}[\ln(x)]$?

Recall $\frac{d}{dx}[f^{-1}(x)] = \frac{1}{f'(f^{-1}(x))}$

$$\frac{d}{dx}[\ln(x)] = \frac{1}{e^{\ln(x)}} = \frac{1}{x}$$

$$\frac{d}{dx}[\log_b(x)] = \frac{d}{dx}\left[\frac{\ln(x)}{\ln(b)}\right] = \frac{1}{\ln(b)} \frac{d}{dx}[\ln(x)] = \frac{1}{\ln(b)} \cdot \frac{1}{x}$$

$$= \frac{1}{x\ln(b)} \text{ or } \frac{1}{\ln(b)x}$$

Find the exact value of each expression.

(a) $\log_2(16) = \log_2(2^4) = 4$

(b) $\log_2\left(\frac{1}{16}\right) = \log_2\left(\frac{1}{2^4}\right) = \log_2(2^{-4}) = -4$

(c) $\log_{64}(2) = y \Rightarrow 2 = 64^y$

64 64

$64 = 2\cdot 2\cdot 2\cdot 2\cdot 2\cdot 2 = 2^6$

$\sqrt[6]{64} = \sqrt[6]{2^6} = 2$

$\log_{64}(2) = \log_{64}\left(64^{\frac{1}{6}}\right)$

$= \frac{1}{6}$!

Review: $a^b a^c = a^{b+c}$ $\log_b(xy) = \log_b(x) + \log_b(y)$

$(a^b)^c = a^{bc}$ $\log_b(x^y) = y\log_b(x)$

$\ln(6x+4) = 3$

$e^{\ln(6x+4)} = e^3$

$6x+4 = e^3$

$6x = e^3 - 4$

$x = \frac{e^3-4}{6}$

$e^{4x-5} = 23$

$4x-5 = \ln(23)$

$4x = \ln(23) + 5$

$x = \frac{\ln(23)+5}{4}$

$\ln(x) + \ln(x-3) = 0$

$e^{\ln(x(x-3))} = e^0$

$\mathcal{D}$:

Need $\underbrace{x > 0 \text{ and } x-3 > 0}_{\boxed{x > 3}}$

$x(x-3) = 1$

$x^2 - 3x = 1$

$x^2 - 3x - 1 = 0$

$a = 1, b = -3, c = -1$

$b^2 - 4ac = 9 - 4(1)(-1) = 13$

$x = \frac{3 \pm \sqrt{13}}{2}$ has a problem.

$\Rightarrow \frac{3 - \sqrt{13}}{2}$ is extraneous. Not in domain!

Solve each equation for x. (Enter your answers as comma-separated lists.)

(a) $e^{2x} - 9e^{x} + 8 = 0$ Let $u = e^x$. Then

$(e^x)^2 = u^2$

$\Rightarrow u^2 - 9u + 8 = 0$

$\Rightarrow (u-8)(u-1) = 0 \Rightarrow$

$u = e^x = 8$ $e^x = 1$

$\Rightarrow x = \ln(8)$ $x = \ln(1) = 0$

If f(x) is increasing, so is its inverse.

$\therefore$ $\ln(x) < 0$ implies

$e^{\ln(x)} < e^0 \Rightarrow$

$\boxed{x < 1}$

LHS < RHS $\Rightarrow$

$e^{LHS} < e^{RHS}$

$\ln(LHS) < \ln(RHS)$