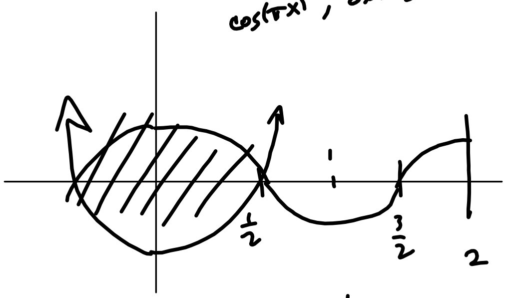
Sketch the region enclosed by the given curves.

$y = 4\cos(\pi x), \quad y = 8x^2 - 2$

$\cos(\pi x)$, $8x^2-2$

cos(x) has period 2π
cos(πx) 2



$8(\frac{1}{2})^2 - 2$
$= 2-2=0$ ✓

$$\text{Area} = \int_{-\frac{1}{2}}^{\frac{1}{2}} \left(4\cos(\pi x) - (8x^2-2)\right) dx$$

$$= 2\left[\int_0^{\frac{1}{2}} 4\cos(\pi x)\,dx - \int_0^{\frac{1}{2}} (8x^2-2)\,dx\right]$$

$$= 2\left[\frac{4}{\pi}\int_0^{\frac{1}{2}} \cos(\pi x)(\pi\,dx) - \left[\frac{8x^3}{3} - 2x\right]_0^{\frac{1}{2}}\right]$$

$$= 2\left[\frac{4}{\pi}\left[\sin(\pi x)\right]_0^{\frac{1}{2}} - \left[\frac{1}{3} - 1\right]\right]$$

$$= \frac{8}{\pi}\left[1-0\right] - 2\left[-\frac{2}{3}\right]$$

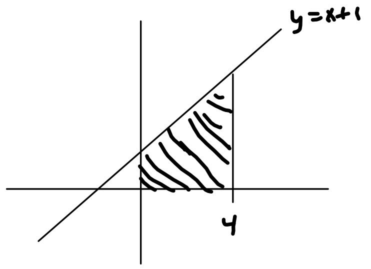
$$= \frac{8}{\pi} + \frac{4}{3}$$ ✓

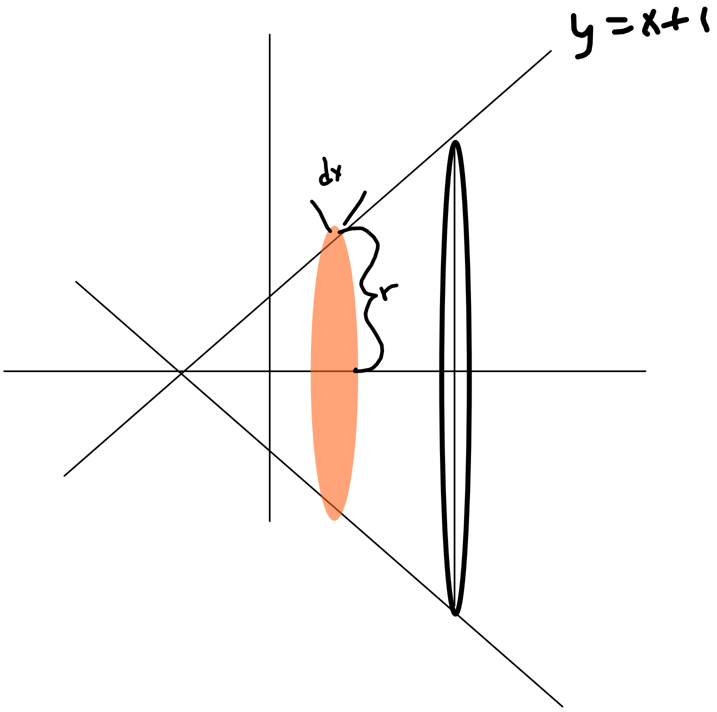
Find the volume V of the solid obtained by rotating the region bounded by the given curves about the specified line.

5.2 #1 $y = x + 1,\ y = 0,\ x = 0,\ x = 4;$ about the x-axis

$V = \frac{7\pi}{3}$ ✗ $\boxed{\frac{124\pi}{3}}$

Sketch the region.





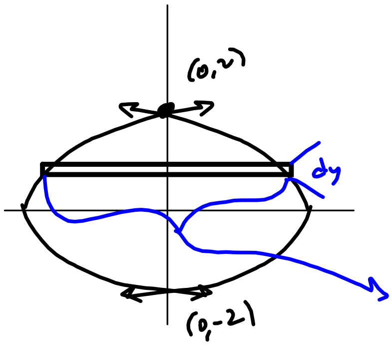
$$\pi \int_a^b f(x)^2\,dx$$

$$= \pi \int_0^4 (x+1)^2\,dx$$

4

Sketch the region enclosed by the given curves. Decide whether to integrate with respect to x or y. Draw a typical approximating rectangle. Find the area of the region.

5.1 #4 $x = 16 - 4y^2, \quad x = 4y^2 - 16$



$4y^2 - 16 - (16 - 4y^2)$

$= 8y^2 - 32$

$$\text{Area} = \int_{-2}^{2} (8y^2 - 32)\,dy = 8 \cdot 2 \int_{0}^{2} (y^2 - 4)\,dy$$

$$= 16 \left[\frac{y^3}{3} - 4y \right]_0^2 =$$

f is 1-to-1 function if

$$[f(x_1)=f(x_2) \Longrightarrow x_1=x_2]$$
$$\Longleftrightarrow$$
$$[x_1\neq x_2 \Longrightarrow f(x_1)\neq f(x_2)]$$

f^{-1} is the function inverse of f.

$$(f^{-1}\circ f)(x)= f^{-1}(f(x)) = x$$
$$(f\circ f^{-1})(x) = f(f^{-1}(x)) = x$$

Domain of $f = \mathcal{D}(f) = A$

Range of $f = \mathcal{R}(f) = B$

$\mathcal{D}(f^{-1}) = \mathcal{R}(f)$

$\mathcal{R}(f^{-1}) = \mathcal{D}(f)$

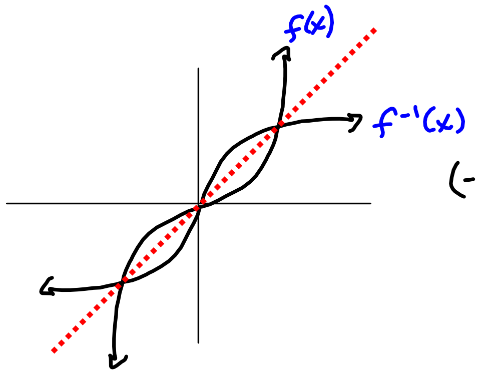
The graph of f^{-1} is obtained by reflection about $y=x$.

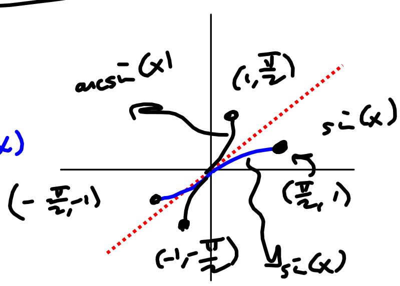
$f(x)=x^3 \Longrightarrow f^{-1}=?$

swap x & y & solve for y:

$y^3=x \Longrightarrow$

$\sqrt[3]{y^3} = \boxed{\sqrt[3]{x} = f^{-1}(x)}$

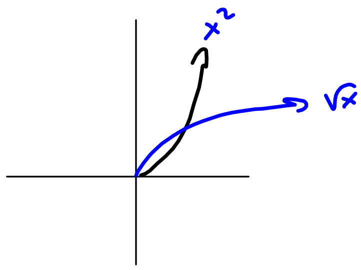




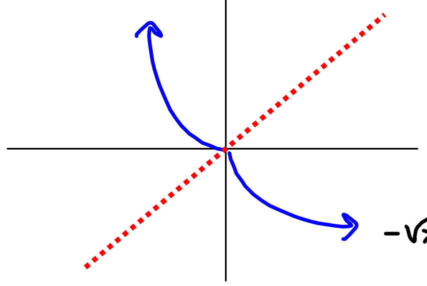
Notice that we had to restrict the domain of sin(x) in order to *make* it 1-to-1.

$f(x)=x^2$ is NOT 1-to-1, but

restricted to $[0,\infty)$ makes it 1-to-1



Restrict x to $(-\infty,0]$ in x^2, then $y=-\sqrt{x}$ is the inverse



Derivative of the inverse formula:

$$(f^{-1})'(x)=\frac{1}{f'(f^{-1}(x))}$$

Derivative of the inverse:

1. **The formula, above, point by point.**
2. **Find the inverse. Take its derivative.**

The advantage of the formula is that you don't *have* to find the inverse function. Just find one or two points.

Find $(f^{-1})'(a)$.

6.1 #16

$f(x) = 3x^3 + 4x^2 + 3x + 9, \quad a = 9$

$$(f^{-1})'(a) = \frac{1}{f'(f^{-1}(a))}$$

Finding the inverse is HARD.

Solve $f(x) = 9$

$3x^3 + 4x^2 + 3x + 9 = 9$

$3x^3 + 4x^2 + 3x = 0$

$x(3x^2 + 4x + 3) = 0 \rightarrow$

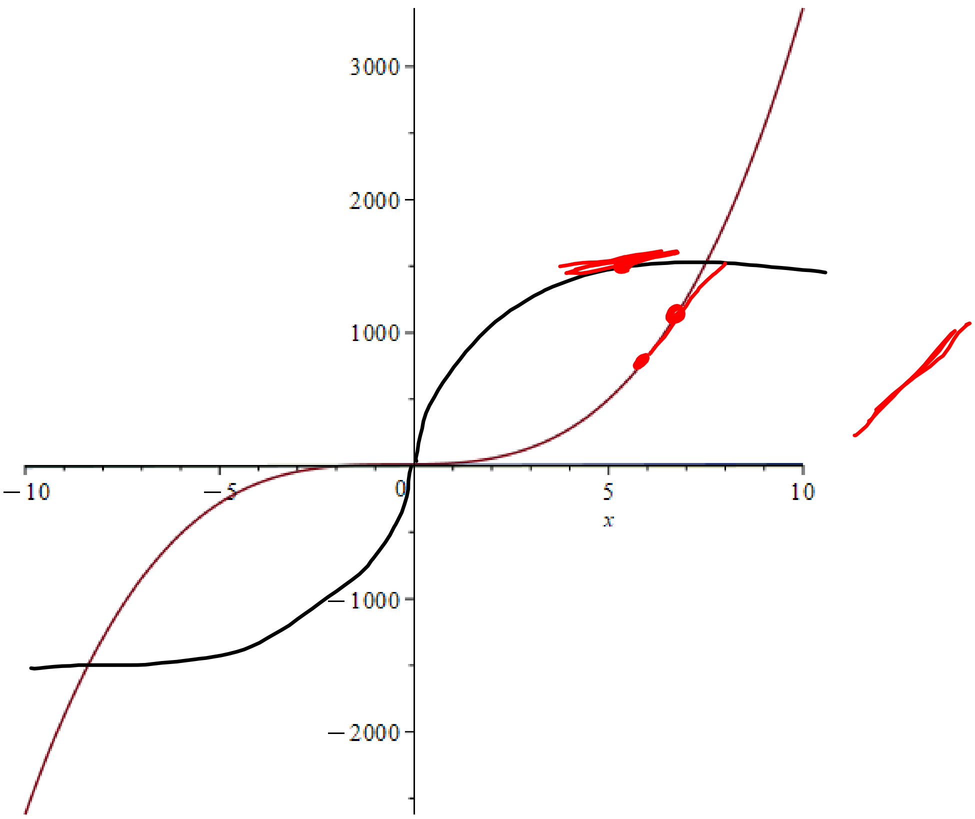
$x = 0$, blind one.

$f^{-1}(9) = 0$

$f'(x) = 9x^2 + 8x + 3$

$f'(0) = 3 \rightarrow$

$$(f^{-1})'(9) = \frac{1}{f'(0)} = \frac{1}{3}$$


3000
2000
1000
−1000
−2000
−10
−5
0
5
10
x