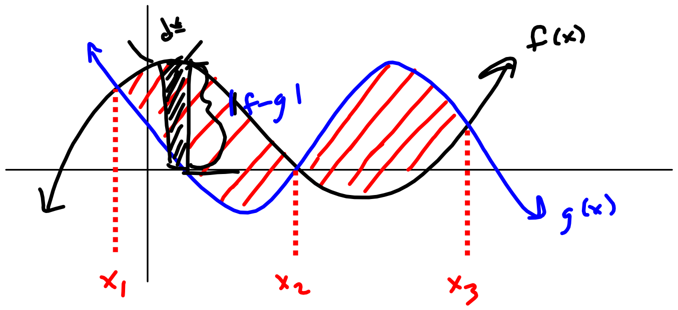
$\int_{x_1}^{x_2} (\text{UPPER} - \text{LOWER})\, dx$ = Area between Upper & lower function.



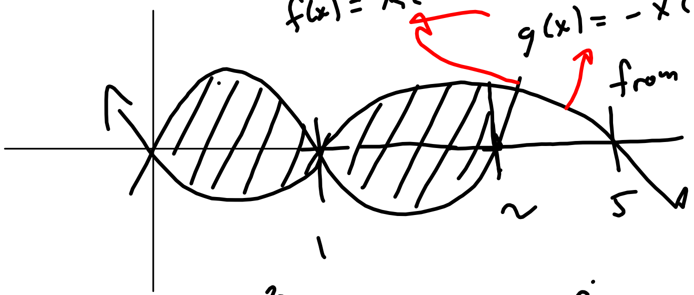
$$\text{Area} = \int_{x_1}^{x_3} |f(x) - g(x)|\,dx = \int_{x_1}^{x_2} (f(x) - g(x))\,dx + \int_{x_2}^{x_3} (g(x) - f(x))\,dx$$

Make it a 5.2 question:

A bit tough with g(x) dipping below the x-axis.

$f(x) = x(x-1)(x-2)$ $g(x) = -x(x-1)(x-5)$

from $x=0$ to $x=2$



Area $= \int_0^2 |f(x) - g(x)|\, dx = \int_0$

$$f := x \to x \cdot (x-1) \cdot (x-2)$$

$$f := x \mapsto x \cdot (x-1) \cdot (x-2)$$

$$g := x \to -x \cdot (x-1) \cdot (x-5)$$

$$g := x \mapsto -x \cdot (x-1) \cdot (x-5)$$

$$\int_0^2 |f(x) - g(x)|\, dx$$

$$4$$

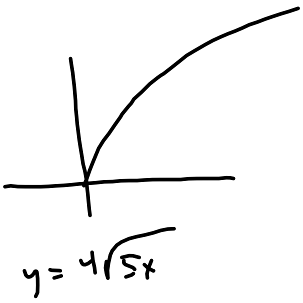
$$\int_0^1 (f(x) - g(x))\, dx + \int_1^2 (g(x) - f(x))\, dx$$

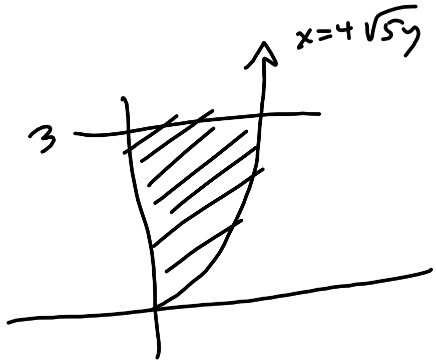
$$4$$

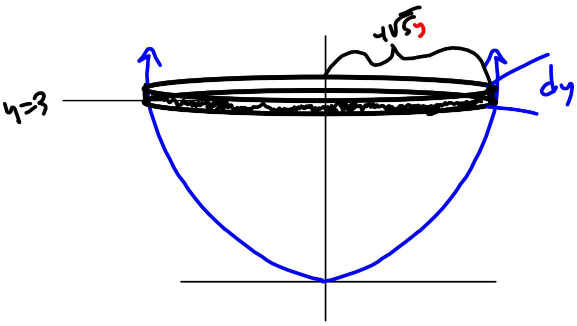
Find the volume V of the solid obtained by rotating the region bounded by the given curves about the specified line.

5.2 #3

$x = 4\sqrt{5y}, \quad x = 0, \quad y = 3; \quad$ about the y-axis







$$\text{Vol} = \pi \int_0^3 \left(4\sqrt{5y}\right)^2 dy$$

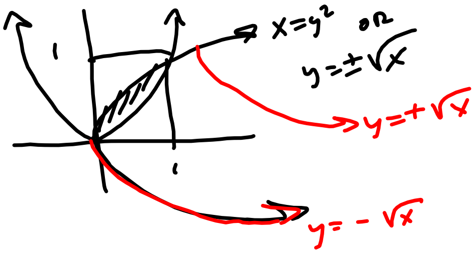
$$= \pi \int_a^b (g(y))^2 \, dy$$

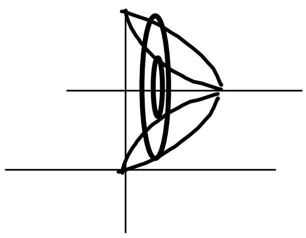
$$= \pi \int_0^3 16 \cdot 5y \, dy = 80\pi \left[\frac{y^2}{2}\right]_0^3$$

$$= 80\pi \left[\frac{9}{2}\right] = 360\pi$$

6 Find the volume V of the solid obtained by rotating the region bounded by the given curves about the specified line.

$y = x^2, \; x = y^2$; about $y = 1$





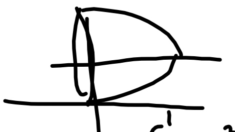
More a washer than a disk.

Strategy:

Subtract small disks from larger disks

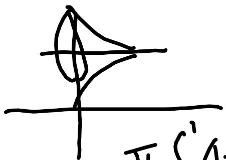
Inner: $1-\sqrt{x}$

Outer: $1-x^2$



$\pi \int_0^1 (1-x^2)^2 dx$

$-$



$\pi \int_0^1 (1-\sqrt{x})^2 dx$

$$\pi \int_0^1 \left((1-x^2)^2 - (1-\sqrt{x})^2 \right) dx$$

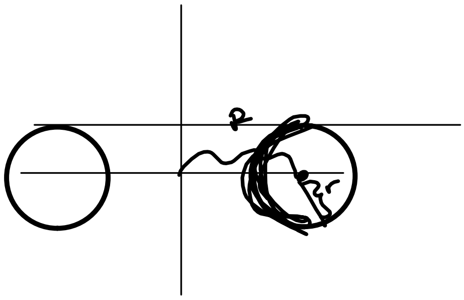
$$= \pi \int_0^1 \left((x^4 - 2x^2 + 1) - (x - 2\sqrt{x} + 1) \right) dx$$

$$= \pi \int_0^1 \left(x^4 - 2x^2 + 1 - x + 2\sqrt{x} - 1 \right) dx = \pi \int_0^1 \left(x^4 - 2x^2 - x + 2x^{\frac{1}{2}} \right) dx$$

$$= \pi \left[\frac{x^5}{5} - \frac{2}{3}x^3 - \frac{x^2}{2} + \frac{4}{3}x^{\frac{3}{2}} \right]_0^1$$

$$= \pi \left[\frac{1}{5} - \frac{2}{3} - \frac{1}{2} + \frac{4}{3} \right] = \pi \left[\frac{1}{5} \cdot \frac{6}{6} - \frac{2}{3} \cdot \frac{10}{10} - \frac{15}{30} + \frac{4}{3} \cdot \frac{10}{10} \right]$$

$$= \pi \left[\frac{6 - 20 - 15 + 40}{30} \right] = \pi \left[\frac{11}{30} \right] = \frac{11\pi}{30}$$



circle

$(x-R)^2 + y^2 = r^2$

solve for x:

$(x-R)^2 = r^2 - y^2$

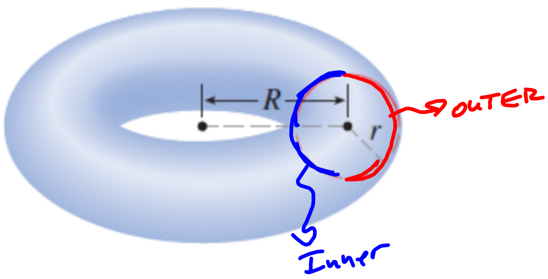
$x - R = \pm\sqrt{r^2 - y^2}$

$x = R \pm \sqrt{r^2 - y^2}$

$R + \sqrt{r^2 - y^2} =$ OUTER

$R - \sqrt{r^2 - y^2} =$ INNER

A *torus* with radii r and R is a donut-shaped solid as shown in the figure.



(a) Set up, but do not evaluate, a definite integral for the volume of the torus.

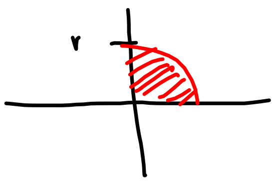
$$V = \pi \int_{-r}^{r} \left(\text{OUTER}^2 - \text{INNER}^2\right) dy$$

$$= \pi \int_{-r}^{r} \left(\left(R + \sqrt{r^2-y^2}\right)^2 - \left(R - \sqrt{r^2-y^2}\right)^2\right) dy$$

$$= 2\pi \int_{0}^{r} \left(\left(R^2 + 2R\sqrt{r^2-y^2} + r^2-y^2\right) - \left(R^2 - 2R\sqrt{r^2-y^2} + r^2-y^2\right)\right) dy$$

$$= 2\pi \int_{0}^{r} 4R\sqrt{r^2-y^2}\, dy = 8\pi R \int_{0}^{r} \sqrt{r^2-y^2}\, dy = 8\pi R \, \frac{\pi r^2}{4}$$

$$= 2\pi^2 R r^2$$



$x = \sqrt{r^2-y^2}$

Right $\frac{1}{4}$-circle of radius r

5.2 #8

$$\text{Pi}\cdot\int_0^{\frac{pi}{4}} \left((5\cdot\cos(x)+1)^2-(5\cdot\sin(x)+1)^2\right)\,dx$$

$$\pi\left(-10+25\sin\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{4}\right)+10\sin\left(\frac{\pi}{4}\right)+10\cos\left(\frac{\pi}{4}\right)\right)$$

$$=\pi\left[-10+25\cdot\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{2}}{2}+10\cdot\frac{\sqrt{2}}{2}+10\cdot\frac{\sqrt{2}}{2}\right]$$

$$=\pi\left[-10+\frac{25}{2}+5\sqrt{2}+5\sqrt{2}\right]=\pi\left[\frac{5}{2}+10\sqrt{2}\right]$$