$$\int_0^{15}\frac{x\,dx}{\sqrt{3x+4}} = \frac{1}{3}\int_0^{15}\frac{x\cdot 3dx}{\sqrt{3x+4}} = \frac{1}{3}\int_{x=0}^{x=15}\frac{x\,du}{\sqrt{u}}$$

$u = 3x+4 \Longrightarrow 3x = u-4 \Longrightarrow x = \frac{u-4}{3}$

$du = 3dx$

$$= \frac{1}{3}\int_{x=0}^{x=15}\frac{\frac{u-4}{3}du}{\sqrt{u}} = \frac{1}{9}\int_{\substack{x=0\\u=4}}^{\substack{u=49\\x=15}}\left(\frac{u}{\sqrt{u}} - \frac{4}{\sqrt{u}}\right)du$$

$$= \frac{1}{9}\int_{x=0}^{x=15}\left(u^{\frac{1}{2}} - 4u^{-\frac{1}{2}}\right)du = \frac{1}{9}\left[\frac{2}{3}u^{\frac{3}{2}} - 8u^{\frac{1}{2}}\right]_{x=0}^{x=15}$$

$$\frac{1}{9}\left[\frac{2}{3}(3x+4)^{\frac{3}{2}} - 8(3x+4)^{-\frac{1}{2}}\right]_0^{15}$$

$$= \frac{1}{9}\left(\left[\frac{2}{3}(3(15)+4)^{\frac{3}{2}} - 8(3(15)+4)^{\frac{1}{2}}\right] - \left[\frac{2}{3}(4)^{\frac{3}{2}} - 8(4)^{\frac{1}{2}}\right]\right)$$

$$= \frac{1}{9}\left(\frac{2}{3}(49)^{\frac{3}{2}} - 8(49)^{\frac{1}{2}}\right) - \left(\frac{2}{3}(8) - 8(2)\right)$$

$$= \frac{1}{9}\left(\frac{2}{3}(7^3) - 8(7)\right) - \left(\frac{16}{3} - \frac{16}{1}\cdot\frac{3}{3}\right)$$

$$= \frac{1}{9}\left(\frac{686}{3} - \frac{56}{1}\cdot\frac{3}{3}\right) - \left(-\frac{32}{3}\right)$$

$$= \frac{1}{9}\left(\frac{518}{3} + \frac{32}{3}\right) = \frac{1}{9}\left(\frac{550}{3}\right) = \boxed{\frac{550}{27} = \int}$$

$49 \times 7 = 343$, $343 \times 2 = 686$

$56 \times 3 = 168$

$686 - 168 = 518$

$x = 0 = \frac{u-4}{3} \Longrightarrow u-4 = 0 \Longrightarrow u = 4$

$x = 15 = \frac{u-4}{3} \Longrightarrow u-4 = 45 \Longrightarrow u = 49$

$$\frac{1}{9}\left[\frac{2}{3}u^{\frac{3}{2}} - 8u^{\frac{1}{2}}\right]_4^{49}$$

$$= \frac{1}{9}\left(\left[\frac{2}{3}(49)^{\frac{3}{2}} - 8(49)^{\frac{1}{2}}\right] - \left[\frac{2}{3}(4)^{\frac{3}{2}} - 8(4)^{\frac{1}{2}}\right]\right)$$

$$= \frac{1}{9}\left(\left[\frac{2}{3}\left[7^3\right] - 8(7)\right] - \left[\frac{2}{3}(8) - 16\right]\right)$$

$$= \frac{1}{9}\left[\left(\frac{686}{3} - \frac{56}{1}\cdot\frac{3}{3}\right) - \left(\frac{16}{3} - \frac{48}{3}\right)\right]$$

$$= \frac{1}{9}\left[\frac{518}{3} + \frac{32}{3}\right] = \frac{550}{27}$$

PT 1

$[a,b]=[1,16]$ $(1\le x\le 16)$

$$\text{Area} \approx \sum_{k=1}^{n} f(x_k)\,\Delta x = \sum_{k=1} f(1+k\Delta x)\,\Delta x$$

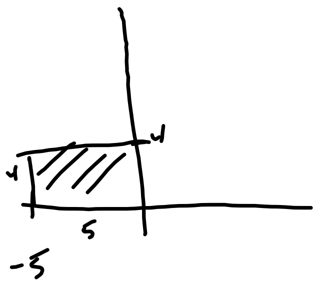
$$= \Delta x \sum f(1+k\Delta x)$$

$$\left(\begin{aligned} \Delta x &= \frac{b-a}{n} = \frac{16-1}{n} = \frac{15}{n} \\ x_k &= a+k\Delta x = 1+\frac{15}{n}k \end{aligned}\right)$$

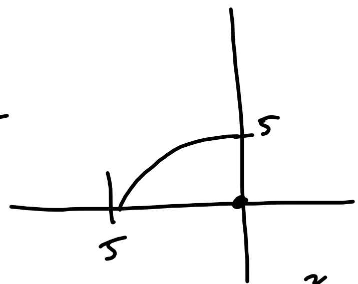
Scratch

$$= \frac{15}{n}\sum_{k=1}^{n}\left(1+\frac{15k}{n}\right)^{\frac{1}{3}}$$

$$\int_{-5}^{0} \left(4 + \sqrt{25-x^2}\right) dx = \int_{-5}^{0} 4\,dx + \int_{-5}^{0} \sqrt{25-x^2}\,dx$$



+



$A_1 = 20$

$A_2 = \frac{\pi (5)^2}{4} = \frac{25\pi}{4} + 20$

Given that $\int_0^1 x^2\,dx = \frac{1}{3}$, use this fact and the properties of definite integrals to evaluate $\int_0^1 (7 - 6x^2)\,dx$.

$$\int_0^1 (7-6x^2)\,dx = \int_0^1 7\,dx - \int_0^1 6x^2\,dx$$

$$= 7\int_0^1 dx - 6\int_0^1 x^2\,dx$$

$$= 7(1) - 6\left(\tfrac{1}{3}\right) \quad = 7-2=5$$

$$\int_{-5}^{2} f(x)\,dx + \int_{2}^{3} f(x)\,dx - \int_{-5}^{-3} f(x)\,dx$$

$$= \int_{-5}^{3} f - \int_{-5}^{-3} f \quad = \int_{-3}^{3} f$$

$$= \int_{-5}^{-3} f + \int_{-3}^{3} f - \int_{-5}^{-3} \quad = \int_{-3}^{3} f$$

9. Find $\int_0^{7} f(x)\,dx$ if

$$f(x) = \begin{cases} 5 & \text{if } x < 5 \\ x & \text{if } x \geq 5. \end{cases}$$

$$\int_0^7 f(x)\,dx = \int_0^5 5\,dx + \int_5^7 x\,dx$$

$$= 5x\Big]_0^5 + \frac{x^2}{2}\Big]_5^7 = 25 + \frac{49}{2} - \frac{25}{2} = \frac{25}{2} + \frac{49}{2} = \frac{74}{2} = \boxed{37}$$

$\frac{d}{dx}\left[\int_1^x t^2 dt\right]$ 2 ways:

#13

FTC I: write x^2

FTC II: $\int_1^x t^2 dt = \frac{t^3}{3}\Big]_1^x = \frac{x^3}{3} - \frac{1}{3}$

$\frac{d}{dx}[\text{previous}] = \frac{3x^2}{3} = x^2$ ✓

Find the derivative of the function.

#14

$$g(x) = \int_{\tan(x)}^{3x^2} \frac{1}{\sqrt{5+t^4}}\, dt$$

$\frac{d}{dx}\left[\int_0^{3x^2} \frac{dt}{\sqrt{t^4+5}}\right] = \frac{6x}{\sqrt{(3x^2)^4+5}}$

$= \int_{\tan x}^{0} \sim + \int_0^{3x^2} = -\int_0^{\tan(x)} \frac{dt}{\sqrt{5+t^4}} + \int_0^{3x^2} \frac{dt}{\sqrt{t^4+5}}$

Differentiate

If $f(x) = \int_0^x \frac{dt}{\sqrt{t^4+5}} \Rightarrow f'(x) = \frac{1}{x^4+5}$

& $f(\tan(x)) = \int_0^{\tan(x)} \frac{dt}{\sqrt{t^4+5}}$

$\frac{d}{dx}[f(\tan(x))] = \frac{df}{d(\tan(x))} \cdot \frac{d(\tan(x))}{dx} = \frac{1}{\tan^4(x)+5} \cdot \sec^2(x)$