$$\int \sec(t)\,(9\sec(t) + 2\tan(t))\,dt$$

$$= \int (9\sec^2(t) + 2\sec(t)\tan(t))\,dt$$

$$= 9\tan(t) + 2\sec(t) + C$$

$$\int \frac{\sin(2x)}{\sin(x)}\,dx = \int \frac{2\sin(x)\cos(x)}{\sin(x)}\,dx$$

$$= \int 2\cos(x)\,dx = 2\sin(x) + C$$

$$\int_{-2}^{1} (x - 8|x|)\,dx$$

$$= \int_{-2}^{0} (x - 8(-x))\,d + \int_{0}^{1} (x - 8x)\,dx$$

1

$$= \int_{-2}^{0} 9x\,dx + \int_{0}^{1} -7x\,dx$$

$$\int_{0}^{4} (4y - y^2)\,dy = \left[\frac{4}{2}y^2 - \frac{1}{3}y^3\right]_{0}^{4}$$

$$= 2(4^2) - \frac{1}{3}(4^3)$$

$$= 32 - \frac{64}{3}$$

$$= \frac{96-64}{3} = \frac{32}{3}$$

Evaluate the indefinite integral. (Use C for the constant of integration.)

$$\int \sin(t)\sqrt{1+\cos(t)}\,dt = \int \sin(t)\sqrt{u}\,\frac{du}{-\sin(t)}$$

$u = 1+\cos(t) \Longrightarrow$

$du = -\sin(t)\,dt$

$\Longrightarrow dt = \dfrac{du}{-\sin(t)}$

$$= -\int \sqrt{u}\,du$$

$$= -\frac{u^{\frac{3}{2}}}{\frac{3}{2}} + C = -\frac{2}{3}u^{\frac{3}{2}} + C$$

Get a pen tablet suitable for taking notes and doing math.

"Remarkable" is pretty awesome, but around $300.

Evaluate the indefinite integral. (Use C for the constant of integration.)

$$\int \frac{a + bx^4}{\sqrt{5ax + bx^5}}\,dx = \int \cancel{(a+bx^4)}(5ax+bx^5)^{-\frac{1}{2}} \frac{du}{5\cancel{(a+bx^4)}}$$

$$= \int u^{-\frac{1}{2}} \cdot \frac{du}{5}$$

$$= \frac{1}{5}\int u^{-\frac{1}{2}}du$$

$$= \frac{2}{5} u^{\frac{1}{2}} + C$$

$$= \frac{2}{5}(5ax+bx^5)^{\frac{1}{2}} + C$$

$u = 5ax + bx^5 \Rightarrow$

$du = (5a + 5bx^4)\,dx = 5(a + bx^4)\,dx$

$\Rightarrow dx = \dfrac{du}{5(a+bx^4)}$

For myself:

$$\int (5ax+bx^5)^{-\frac{1}{2}}(a+bx^4)\,dx$$

$$= \frac{1}{5}\int \underbrace{(5ax+bx^5)}_{u}{}^{-\frac{1}{2}}\underbrace{(5a+5bx^4)\,dx}_{du}$$

$$= \frac{1}{5}\frac{(5ax+bx^5)^{\frac{1}{2}}}{\frac{1}{2}} + C = \frac{2}{5}(5ax+bx^5)^{\frac{1}{2}} + C$$

$$\int \frac{\cos\left(\frac{\pi}{x^9}\right)}{x^{10}}\,dx = \frac{1}{-9\pi}\int \cos(\pi x^{-9})\,(-9\pi x^{-10}\,dx)$$

$u = \frac{\pi}{x^9} = \pi x^{-9} \Rightarrow$

$du = -9\pi x^{-10}\,dx$

$$= -\frac{1}{9\pi}\int \cos(u)\,du$$

$$= -\frac{1}{9\pi}\sin(u) + C$$

$$= -\frac{1}{9\pi}\sin\left(\frac{\pi}{x^9}\right) + C$$

Evaluate the indefinite integral. (Use C for the constant of integration.)

$$\int x(4x+5)^8\,dx = \int x\, u^8\left(\frac{du}{4}\right) = \frac{1}{4}\int x\, u^8\,du$$

$u = 4x+5 \Rightarrow u-5 = 4x \Rightarrow x = \frac{u-5}{4} \leftarrow$?!

$du = 4\,dx$

$dx = \frac{du}{4}$

$$= \frac{1}{4}\int \frac{u-5}{4}\,u^8\,du$$

$$= \frac{1}{16}\int u^8(u-5)\,du$$

$$= \frac{1}{16}\int (u^9 - 5u^8)\,du = \frac{1}{16}\left[\frac{u^{10}}{10} - \frac{5u^9}{9}\right] + C$$

$$= \frac{1}{16}\left[\frac{1}{10}(4x+5)^{10} - \frac{5}{9}u^9\right] + C$$

Evaluate the indefinite integral. (Use C for the constant of integration.)

$$\int x^3\sqrt{x^2+40}\,dx = \int x^3\sqrt{u}\,\frac{du}{2x}$$

$u = x^2+40$

$du = 2x\,dx$

$dx = \frac{du}{2x}$

$$= \frac{1}{2}\int x^2\sqrt{u}\,du \Rightarrow$$

$u = x^2+40$

$x^2 = u-40$

$$= \frac{1}{2}\int (u-40)u^{\frac{1}{2}}\,du, \text{ etc.}$$

Evaluate the definite integral.

$$\int_0^{\pi/2} 5\cos(x)\sin(\sin(x))\,dx = 5\int_0^{\frac{\pi}{2}} \sin(\sin(x))\cos(x)\,dx$$

$u = \sin(x)$

$du = \cos(x)\,dx$

$$= 5\int_{x=0}^{x=\frac{\pi}{2}} \sin(u)\,du$$ * SEE BELOW

$$= 5\left[-\cos(u)\right]_{x=0}^{x=\frac{\pi}{2}} = 5\left[-\cos(\sin(x))\right]_0^{\frac{\pi}{2}} = -5\left[\cos(1) - \cos(0)\right]$$

$$= -5\left[\cos(1) - 1\right] = 5 - 5\cos(1)$$

$$5\int_{x=0}^{x=\frac{\pi}{2}} \sin(u)\,du = 5\int_{0=u}^{1=u} \sin(u)\,du = -5\cos(u)\Big]_0^1 = -5\cos(1) - (-5)\cos(0)$$

$$= 5 - 5\cos(1)$$

$u = \sin(x)$

$\sin\left(\frac{\pi}{2}\right) = 1$

$\sin(0) = 0$

Swap / Replace the limits of integration to something in terms of u

This is a (the) other way to handle the limits of integration in a *u*-substitution exercise.