4 The Substitution Rule If $u = g(x)$ is a differentiable function whose range is an interval I and f is continuous on I, then

$$\int f(g(x))\,g'(x)\,dx = \underline{\int f(u)\,du}$$

If $g(x) = u$

Then $dg = du = g'(x)\,dx$

$$\int\left(\sqrt{x^2+7}\right)(2x\,dx) = \int \sqrt{u}\,du = \int u^{\frac{1}{2}}\,du = \frac{u^{\frac{3}{2}}}{\frac{3}{2}} + C$$

Let $u = x^2+7 \Rightarrow$ $\quad = \frac{2}{3}(x^2+7)^{\frac{3}{2}} + C$

$du = 2x\,dx$

$$\int_0^1 (u+8)(u-9)\,du = \int_0^1 (u^2 - u - 9)\,du \quad \to 72$$

$$= \int_0^1 u^2\,du - \int_0^1 u\,du - \int_0^1 9\,du \quad \to 72$$

$$= \frac{u^3}{3}\Big]_0^1 - \frac{u^2}{2}\Big]_0^1 - 9u\Big]_0^1 \quad \to -72u$$

$72 \times 6 = 432$

$$= \left[\frac{u^3}{3} - \frac{u^2}{2} - 9u\right]_0^1 = \frac{1}{3} - \frac{1}{2} - 9 - [0 - 0 - 0] \quad (9 \to 72)$$

$$-72u \quad = \frac{2 - 3 - 54}{6} \quad (54 \to 432) \quad = -\frac{433}{6}$$

2 Evaluate the integral by making the given substitution. (Use C for the constant of integration.)

$$\int x^2\sqrt{x^3+27}\,dx, \quad u = x^3 + 27$$

$u = x^3 + 27$ $\rightarrow$ $\frac{1}{3}\int (x^3+27)^{\frac{1}{2}}(3x^2dx) = \frac{1}{3}\int u^{\frac{1}{2}}du$

$du = 3x^2dx$ $= \frac{1}{3}(x^3+27)^{\frac{3}{2}}/(3/2) + C$

$\rightarrow dx = \frac{du}{3x^2}$

$\rightarrow \int x^2\sqrt{x^3+27}\,dx = \int x^2\sqrt{u}\,\frac{du}{3x^2} = \int \sqrt{u}\,\frac{du}{3} =$

$= \frac{1}{3}\int u^{\frac{1}{2}}du = \frac{1}{3}\left[\frac{u^{3/2}}{\frac{3}{2}}\right] + C$

$= \frac{2}{9}(x^3+27)^{\frac{3}{2}} + C$

$\int (7-5x)^6\,dx = -\frac{1}{5}\int (7-5x)^6(-5dx)$

$\int \sin(5x)\cos(x)\,dx = \frac{1}{5}\int \sin$

oops!

$\cos(x)$ is NOT $\frac{d}{dx}[\sin(5x)]$

This last one isn't set up for *u*-substitution. It requires something more subtle that I haven't shown you.

$$\sin u \cos v = \frac{1}{2}[\sin(u+v) + \sin(u-v)]$$

Click here for College Trigonometry Cheat Sheet.

$= \frac{1}{2}\int (\sin(6x) + \sin(4x))\,dx = \frac{1}{2}\left[-\frac{1}{6}\cos(6x) - \frac{1}{4}\cos(4x)\right] + C$

https://harryzaims.com/public_html/122/1420-fall-23/notes/cheat-sheet-test-4.pdf

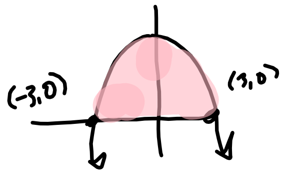
4.3 #13

$$\int_{-3}^{3}(9-x^2)\,dx$$

$$= 2\int_0^3 (9-x^2)\,dx$$

$$= 2\left[9x-\frac{x^3}{3}\right]_0^3 = 2\left[9(3)-\frac{3^3}{3}\right]-[0-0]$$

$$= 2[27-9] = 2[18] = 36$$



Find the derivative of the function.

$$g(x)=\int_{6x}^{7x}\frac{u^2-5}{u^2+5}\,du \qquad \left[Hint: \int_{6x}^{7x} f(u)\,du = \int_{6x}^{0} f(u)\,du + \int_0^{7x} f(u)\,du\right]$$

$$= -\int_0^{6x} \sim + \int_0^{7x} =$$

$$\to g'(x) = -\left(\frac{(6x)^2-5}{(6x)^2+5}\right)(6) + \left(\frac{(7x)^2-5}{(7x)^2+5}\right)(7)$$

Don't Over-think it, dude!

OR

$$= -\int_0^{6x}\sim + \int_0^{7x}\sim = -\int_0^{6x}\sim + \int_0^{6x+x}\sim$$

$$= -\int_0^{6x}\sim + \int_0^{x}\sim + \int_x^{6x}\sim$$

$$\int_0^5 = \int_0^3 + \int_3^5$$

$$= -\int_0^{6x}\sim + \int_0^{6x}\sim + \int_{6x}^{x}$$

$=$

4.3 #5

Evaluate the integral.

$$\int_1^9 \frac{6 + x^2}{\sqrt{x}}\, dx$$

$$= \int_1^9 \left(\frac{6}{x^{\frac{1}{2}}} + \frac{x^2}{x^{\frac{1}{2}}}\right) dx$$

$$= \int_1^9 \left(6x^{-\frac{1}{2}} + x^{\frac{3}{2}}\right) dx = \text{etc.}$$

Verify by differentiation whether the following formula is true or false.

4.4 #2

$$\int \cos^2(x)\, dx = \frac{1}{2}x + \frac{1}{4}\sin(2x) + C$$

$$\frac{d}{dx}\left[\frac{1}{2}x + \frac{1}{4}\sin(2x)\right] =$$

$$\frac{1}{2} + \frac{1}{2}\cos(2x)$$

$$= \frac{1}{2} + \frac{1}{2}\left[2\cos^2(x) - 1\right]$$

$$= \frac{1}{2} + \cos^2 x - \frac{1}{2} = \cos^2(x) \quad \text{Yes!}$$

$$\cos^2(x) = \frac{\cos(2x) + 1}{2}$$

$$\longrightarrow \int \cos^2(x)\, dx = \frac{1}{2}\int (\cos(2x) + 1)\, dx$$

$$= \frac{1}{2}\left[\frac{1}{2}\sin(2x) + x\right] + C$$

$$= \frac{1}{4}\sin(2x) + \frac{1}{2}x + C$$

$\int$ 4.4 #14b

$$\int_{-2}^{1} (x - 8|x|)\, dx = \int_{-2}^{0} (x + 8x)\, dx + \int_{0}^{1} (x - 8x)\, dx$$