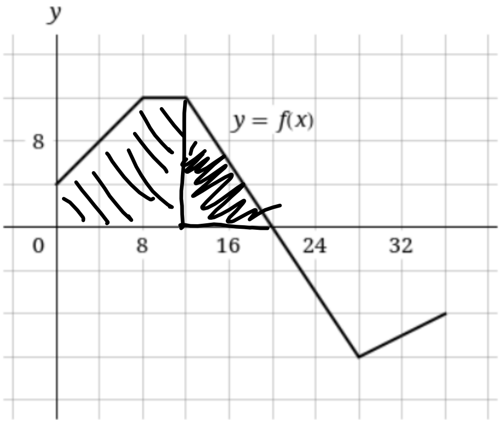


(a) $\int_0^8 f(x)\,dx = 64$

(b) $\int_0^{20} f(x)\,dx = 64+48 = 112$

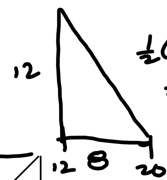
(c) $\int_{20}^{28} f(x)\,dx = -64$

(d) $\int_{12}^{28} f(x)\,dx = 48-64 = -16$

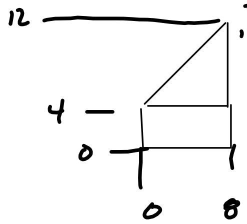
(e) $\int_{12}^{28} |f(x)|\,dx = 48+64 = 112$

(f) $\int_8^0 f(x)\,dx = -64$

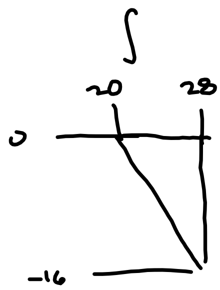
dx is negative.



$\frac{1}{2}(8)(12)$
$= 48$



$= \frac{1}{2}(8)(8)$
$+ 8(4)$
$= 32+32 = 64$



$\frac{1}{2}(8)(-16)$
$= -64$

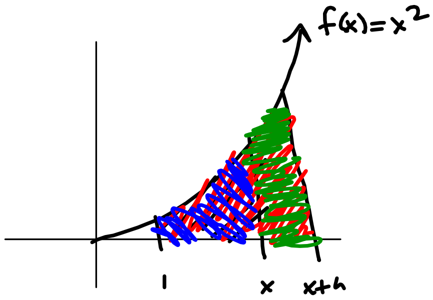
The Fundamental Theorem of Calculus, Part 1 If f is continuous on $[a, b]$, then the function g defined by

$$g(x) = \int_a^x f(t)\,dt \qquad a \leqslant x \leqslant b$$

is continuous on $[a, b]$ and differentiable on (a, b), and $g'(x) = f(x)$.

$\int_2^x f(t)\,dt$ is a function of x.

$\int_1^x t^2\,dt$



$\int_1^x t^2\,dt$ is the area under t^2 from $t=1$ to $t=x$.

$$\frac{d}{dx}\left[\int_1^x t^2\,dt\right] = \lim_{h\to 0} \frac{\int_1^{x+h} t^2\,dt - \int_1^x t^2\,dt}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{\int_1^x t^2\,dt} + \int_x^{x+h} t^2\,dt - \cancel{\int_1^x t^2\,dt}}{h} = \lim_{h\to 0} \frac{\int_x^{x+h} t^2\,dt}{h}$$

Let $x=2$

Consider

$$\frac{\int_2^{2+h} t^2\,dt}{h}$$

Let u such that $\max\limits_{[2,2+h)}\{t^2\} = u^2$

v such that $\max\limits_{[2,2+h]}\{t^2\} = v^2$

Then $v^2(h) \le \int_2^{2+h} t^2\,dt \le u^2(h)$

Assume $h>0$ $\longrightarrow$

$$v^2 \le \frac{\int_2^{2+h} t^2\,dt}{h} \le u^2 \quad \xrightarrow{h\to 0}$$

By continuity of $f(t)=t^2$

$$2^2 \le \lim_{h\to 0} \frac{\int_2^{2+h} t^2\,dt}{h} \le 2^2$$

Squeeze Theorem $\longrightarrow$

$$\lim_{h\to 0} \frac{\int_1^x t^2\,dt}{h} = \frac{d}{dx}\left[\int_1^x t^2\,dt\right] = t^2$$

Derivative of the integral is the integrand.

The Fundamental Theorem of Calculus, Part 2 If f is continuous on $[a, b]$, then

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

where F is any antiderivative of f, that is, a function F such that $F' = f$.

$$\int t^2\,dt = \frac{t^3}{3} + C$$

$$\int_0^3 t^2\,dt = \int_0^3 x^2\,dx = \left.\frac{x^3}{3}\right]_0^3 = \frac{3^3}{3} - \frac{0^3}{3} = 9$$

$[a,b] = [0,3]$

$$\int_0^3 t^2\,dt \qquad \frac{b-a}{n} = \frac{3}{n}$$

$$x_k = 0 + k\left(\frac{3}{n}\right) = \frac{3k}{n}$$

$$\Delta x \sum_{k=1}^{n} f(x_k) = \frac{3}{n}\sum_{k=1}^{n}\left(\frac{3k}{n}\right)^2 = \frac{27}{n^3}\sum_{k=1}^{n} k^2 = \frac{27}{n^3}\left(\frac{n^3 + \cdots}{3}\right)$$

$\xrightarrow{n \to \infty} 9$ See? Same. But harder.

Whole list of antiderivatives, so far.

$$\int cf(x)\,dx = c\int f(x)\,dx \qquad \int [f(x)+g(x)]\,dx = \int f(x)\,dx + \int g(x)\,dx$$

$$\int k\,dx = kx + C \qquad \int x^n\,dx = \frac{x^{n+1}}{n+1} + C \quad (n \neq -1)$$

$$\int \sin x\,dx = -\cos x + C \qquad \int \cos x\,dx = \sin x + C$$

$$\int \sec^2 x\,dx = \tan x + C \qquad \int \csc^2 x\,dx = -\cot x + C$$

$$\int \sec x \tan x\,dx = \sec x + C \qquad \int \csc x \cot x\,dx = -\csc x + C$$

A graphing calculator is recommended.

What is wrong with the equation?

$$\int_{-3}^{1} x^{-5}\,dx = \left.\frac{x^{-4}}{-4}\right]_{-3}^{1} = -\frac{20}{81}$$

$$= -\frac{1}{4} - \frac{(-3)^{-4}}{-4} = -\frac{1}{4} + \frac{1}{81(4)}$$

$$= \frac{-81+1}{4(81)} = \frac{-80}{4(81)} = -\frac{20}{81}$$

Integrand $\frac{1}{x^5}$ blows up @ $x=0 \in [-3,1]$

$$g(x) = \int_1^x \frac{\sin(t^2-7)\cos(t^3)}{t^{\cos(t)}}\,dt$$

$$\Rightarrow g'(x) = \frac{\sin(x^2-2)\cos(x^3)}{x^{\cos(x)}}$$

CHAIN RULE VERSION

$$h(x) = g(\sin(x)) = \int_{1}^{\sin(x)} \frac{\sin(t^2-2)\cos(t^3)}{t^{\cos(t)}}\,dt$$

$$= g(f(x)) \quad (\text{i.e., } f(x) = \sin(x))$$

$$\Rightarrow \frac{dh}{dx} = \frac{dg}{df} \cdot \frac{df}{dx} = \frac{\sin(\sin^2(x)-2)\cos(\sin^3(x))}{\sin(x)^{\cos(\sin(x))}} \cdot \cos(x)$$

$$\frac{dg}{df} \qquad \cdot \frac{df}{dx}$$

Net Change Theorem The integral of a rate of change is the net change:

$$\int_a^b F'(x)\,dx = F(b) - F(a)$$