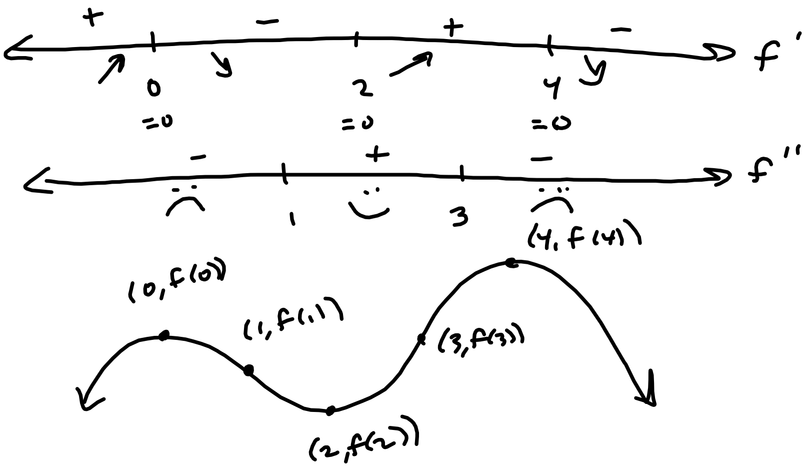
Sketch the graph of a function that satisfies all of the given conditions.

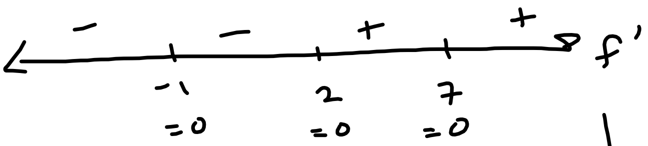
#18

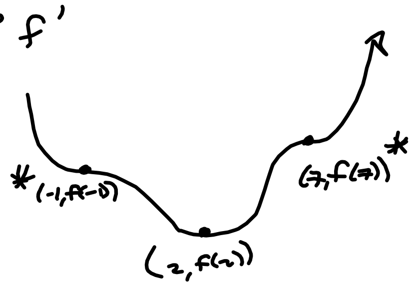
$f'(0) = f'(2) = f'(4) = 0,$
$f'(x) > 0$ if $x < 0$ or $2 < x < 4,$
$f'(x) < 0$ if $0 < x < 2$ or $x > 4,$
$f''(x) > 0$ if $1 < x < 3,$
$f''(x) < 0$ if $x < 1$ or $x > 3$



Suppose the derivative of a function f is $f'(x) = (x + 1)^4(x - 2)^5(x - 7)^6$. On what interval is f increasing? (Enter your answer in interval notation.)

$f'(x) = (x+1)^4 (x-2)^5 (x-7)^6 \quad = x^{15} +$ smaller degree . . .

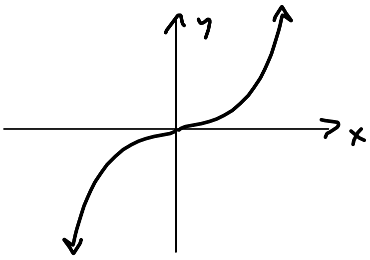




$\boxed{f \text{ increasing on } (2, \infty)}$

* Terrace Point.

#20 $g(x) = 4x|x|. = \begin{cases} 4x^2, & \text{if } x \geq 0 \\ -4x^2, & \text{if } x < 0 \end{cases}$



I.P. : $(0,0)$

$$f'(x) = \begin{cases} 8x & \text{if } x > 0 \\ 0 & \text{if } x = 0 \\ -8x & \text{if } x < 0 \end{cases} = \begin{cases} 8x & \text{if } x \geq 0 \\ -8x & \text{if } x < 0 \end{cases}$$

$f'_+(0) = 8(0) = 0$

$f'_-(0) = -8(0) = 0$

$$\lim_{x\to\infty}\left(\sqrt{9x^2+x}-3x\right):$$

$$\frac{\left(\sqrt{9x^2+x}-3x\right)\left(\sqrt{9x^2+x}+3x\right)}{\sqrt{9x^2+x}+3x}$$

$$=\frac{9x^2+x-9x^2}{\sqrt{9x^2+x}+3x}=\frac{x}{\sqrt{9x^2+x}+3x}$$

$$=\frac{x}{\sqrt{x^2\left(9+\frac{1}{x}\right)}+3x}=\frac{x}{|x|\sqrt{9+\frac{1}{x}}+3x}$$

$$\overset{*}{=}\frac{x}{x\sqrt{9+\frac{1}{x}}+3x}=\frac{x}{x\left(\sqrt{9+\frac{1}{x}}+3\right)}$$

$$=\frac{1}{\sqrt{9+\frac{1}{x}}+3}\xrightarrow{x\to\infty}\frac{1}{\sqrt{9}+3}=\frac{1}{3+3}=\frac{1}{6}$$

$x\neq 0$

$*\ x\to\infty \Rightarrow |x|=x$

$$\lim_{x\to-\infty}\left(\sqrt{9x^2+x}+3x\right):$$

$$=\frac{\left(\sqrt{9x^2+x}+3x\right)\left(\sqrt{9x^2+x}-3x\right)}{\sqrt{9x^2+x}-3x}$$

$$=\frac{9x^2+x-9x^2}{\sqrt{9x^2+x}-3x}=\frac{x}{\sqrt{x^2\left(9+\frac{1}{x}\right)}-3x}$$

$$=\frac{x}{|x|\sqrt{9+\frac{1}{x}}-3x}\overset{*}{=}\frac{x}{-x\sqrt{9+\frac{1}{x}}-3x}$$

$x\to-\infty \Rightarrow x<0 \Rightarrow |x|=-x$

$$=\frac{x}{-x\left(\sqrt{9+\frac{1}{x}}+3\right)}=\frac{1}{-\left(\sqrt{9+\frac{1}{x}}+3\right)}=\frac{1}{-(3+3)}=-\frac{1}{6}$$

$(x\neq 0)$

$$\frac{2x^2+x-2}{x^2+x-6} = \frac{2\left(x-\left(\frac{1+\sqrt{17}}{2}\right)\right)\left(x-\left(\frac{1-\sqrt{17}}{2}\right)\right)}{(x+3)(x-2)}$$

$2x^2+x-2=0$

$b^2-4ac = 1^2-4(2)(-2)$

$=1+16=17$

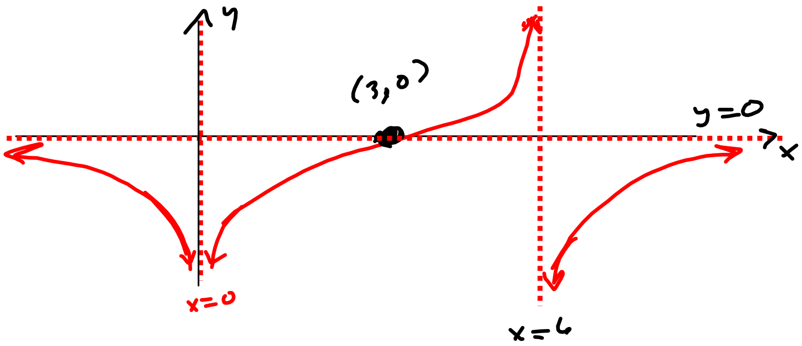
$x = \frac{-1\pm\sqrt{17}}{2(2)} = \frac{-1\pm\sqrt{17}}{4}$

V.A.: $x=-3,\ x=2$

H.A: $y=2$

Find a formula for a function f that satisfies the following conditions.

$$\lim_{x\to\pm\infty} f(x) = 0, \quad \lim_{x\to 0} f(x) = -\infty, \quad f(3) = 0,$$
$$\lim_{x\to 6^-} f(x) = \infty, \quad \lim_{x\to 6^+} f(x) = -\infty,$$



$$-\frac{x-3}{x^2(x-6)}$$

Find how big x must be to make

$$\frac{1}{\sqrt{x}} < .01 = \frac{1}{100}$$

$$\Rightarrow 100 < \sqrt{x}$$

$$\Rightarrow 100^2 < \sqrt{x}^2 = x$$

$$\boxed{10000 < x}$$

Prove $\frac{1}{\sqrt{x}} \xrightarrow{x \to \infty} 0$

Let $\varepsilon > 0$. We find N such that

$\frac{1}{\sqrt{x}} < \varepsilon$ for all $x > N$

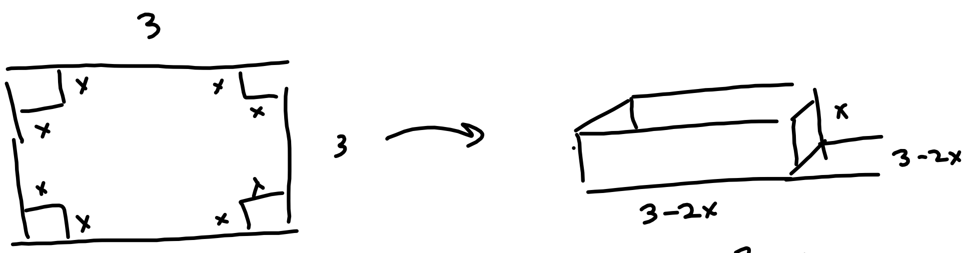
$$\frac{1}{\varepsilon} < \sqrt{x}$$

$$\left(\frac{1}{\varepsilon}\right)^2 < x$$

Take $x > \frac{1}{\varepsilon^2} \equiv N$

<u>Proof</u> Define $N = \frac{1}{\varepsilon^2}$. Then $\forall\, x > N$, we have

$$\left|\frac{1}{\sqrt{x}} - 0\right| = \frac{1}{\sqrt{x}} < \frac{1}{\sqrt{N}} = \frac{1}{\sqrt{\frac{1}{\varepsilon^2}}} = \frac{\sqrt{\varepsilon^2}}{\sqrt{1}} = |\varepsilon| = \varepsilon$$



$V = (3-2x)^2(x)$ to be maximized