Practice Test 3

Find critical #s:

$g(y) = \frac{y-2}{y^2-2y+4} = \frac{y-2}{(y-2)^2} = \frac{1}{y-2} = (y-2)^{-1} \Rightarrow$

$y \neq 2$

$\mathcal{D}: y^2 - 2y + 4 \neq 0$

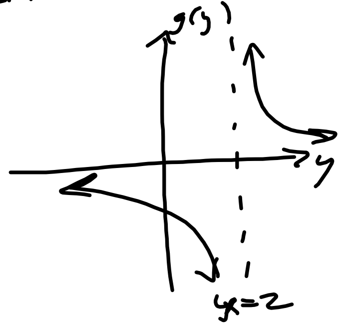
$(y-2)^2 \neq 0$

$y \neq 2.$

$g'(y) = -(y-2)^{-2} = \frac{-1}{(y-2)^2}$

No critical #s.

$\frac{1}{y-2}$



$f(t) = t^{\frac{3}{4}} - 9t^{\frac{1}{4}} \quad \mathcal{D} = [0, \infty)$

$\Rightarrow f'(t) = \frac{3}{4}t^{-\frac{1}{4}} - \frac{9}{4}t^{-\frac{3}{4}} =$

$= \frac{3}{4}t^{-\frac{3}{4}}(t^{\frac{1}{2}} - 3) \quad \underline{\text{SET}} = 0 \Rightarrow t^{\frac{1}{2}} = 3 \Rightarrow \boxed{t = 9}$

$t^{\frac{1}{4}}(t^{\frac{1}{2}} - 9)$

$\nexists$ $t = 0$ makes it $\nexists$

$-\frac{1}{4} - (-\frac{3}{4}) = \frac{1}{2} \quad \frac{3}{4} - \frac{1}{4} = \frac{1}{2}$

$t = 0, 9$ are critical #s.

Note $t = 0, 9 \in \mathcal{D}(f)$

$f(\theta) = 32\theta - 8\tan\theta$

$\theta =$

$f'(\theta) = 32 - 8\sec^2\theta \stackrel{\text{SET}}{=} 0$

& set = undefined.

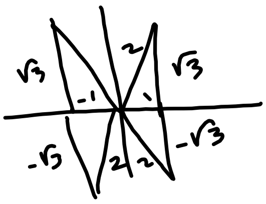
#6

$8\sec^2\theta = 32$

$\sec^2\theta = 4$

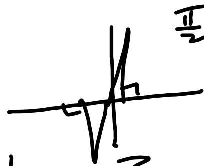
$\cos^2\theta = \frac{1}{4}$

$\cos\theta = \pm\frac{1}{2}$



Can't have $\cos\theta = 0$

$\frac{\sin\theta}{\cos\theta}$



$\frac{\pi}{2}, \frac{3\pi}{2}$

$\theta = \mathbb{R} \setminus \{\frac{\pi}{2} + n\pi \mid n \in \mathbb{Z}\}$

$= \bigcup_{n \in \mathbb{Z}} (\frac{\pi}{2} + n\pi, \frac{\pi}{2} + (n+1)\pi)$

ADVANCED.

$\theta = \frac{\pi}{3} + 2n\pi, \frac{2\pi}{3} + 2n\pi, \frac{4\pi}{3} + 2n\pi, \frac{5\pi}{3} + 2n\pi$

$\theta = \frac{\pi}{3} + n\pi, \frac{2\pi}{3} + n\pi$

$f(x) = -3x^3 + 81x + 7$ Find Absolute Max & min on $[0,4]$.

Extreme Value Theorem

$$
\begin{array}{r|rrrr}
0 & -3 & 0 & 81 & 7 \\
 & & 0 & 0 & 0 \\
\hline
 & -3 & 0 & 81 & \boxed{7 = f(0)}
\end{array}
$$
Abs MIN

$$
\begin{array}{r|rrrr}
4 & -3 & 0 & 81 & 7 \\
 & & -12 & -48 & 132 \\
\hline
 & -3 & -12 & 33 & \boxed{139 = f(4)}
\end{array}
$$

$f'(x) = -9x^2 + 81 \overset{\text{SET}}{=} 0$

$9x^2 = 81$

$x^2 = 9$

$x = \pm 3$

$f(3) =$

$$
\begin{array}{r|rrrr}
3 & -3 & 0 & 81 & 7 \\
 & & -9 & -27 & 162 \\
\hline
 & -3 & -9 & 54 & \boxed{169 = f(3)}
\end{array}
$$
Abs Max

#10
$f(x) = 4x^2 - 3x + 2$ on $[0,2]$ MVT. Find c

① f is polynomial $\Rightarrow \mathcal{D} = \mathbb{R}$ & f is cnts & difbl $\forall x \in \mathbb{R}$, in particular, f is cnts on $[0,2]$ & difbl on $(0,2)$

$$m_{AVG} = \frac{f(2) - f(0)}{2 - 0} = \frac{12 - 2}{2} = \frac{10}{2} = 5$$

$$\begin{array}{r|rrr} 2 & 4 & -3 & 2 \\ & & 8 & 10 \\ \hline & 4 & 5 & 12 \end{array}$$

$$f'(x) = 8x - 3 \overset{SET}{=} m_{AVG} = 5$$

$8x = 8$

$x = \boxed{1 = c}$

MVT for $f(x) = \sqrt{x} = x^{\frac{1}{2}}$ on $[0,16]$

$\sqrt{x}$ dfbl on $(0,\infty) \supseteq (0,16)$

$\sqrt{x}$ cnts on $[0,\infty) \supseteq [0,16]$

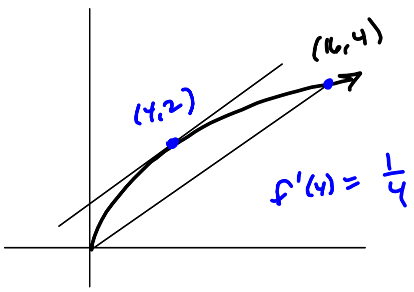
$$m_{AVG} = \frac{f(16) - f(0)}{16 - 0} = \frac{4-0}{16} = \frac{1}{4}$$

$$f'(x) = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}} \overset{SET}{=} \frac{1}{4}$$

$$2\sqrt{x} = 4$$

$$\sqrt{x} = 2$$

$$\boxed{x = 4}$$

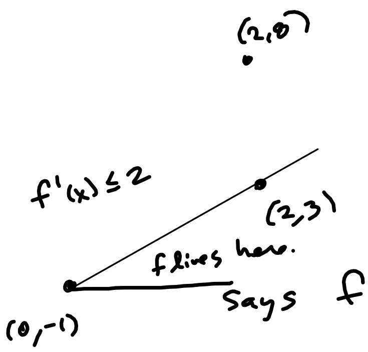


Racetrack principle.

$f(0) = -1$, $f(2) = 8$, & $f'(x) \le 2 \ \forall x$?

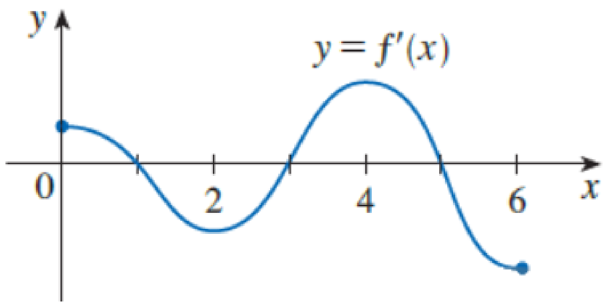
$f(2)$: $f(2) \le f(0) + 2(2-0) = -1 + 4 = 3$

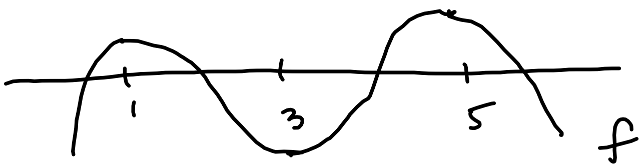
So $f(2) = 8$ is impossible.



Says f is under that line.

The graph of the *derivative* f' of a function f is shown.





Inc: $(0,1) \cup (3,5)$
Dec: $(1,3) \cup (5,6)$

Consider the following. (If an answer does not exist, enter DNE.)

$f(x) = 7\sin(x) + 7\cos(x), \quad 0 \le x \le 2\pi$

(a) Find the interval on which f is increasing. (Enter your answer using interval notation.)

$\left(0, \frac{\pi}{4}\right), \left(\frac{5\pi}{4}, 2\pi\right)$

Find the interval on which f is decreasing. (Enter your answer using interval notation.)

$\left(\frac{\pi}{4}, \frac{5\pi}{4}\right)$

(b) Find the local maximum and minimum values of f.

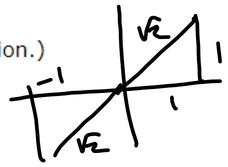
local minimum value $-7\sqrt{2}$

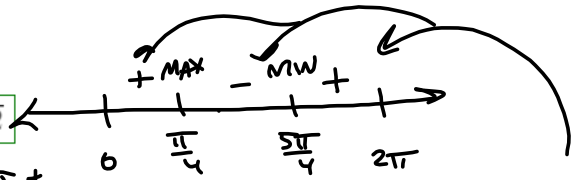
$f'(x) = 7\cos(x) - 7\sin(x)$ SET $= 0$

$\Rightarrow \cos(x) = \sin(x)$

$\tan(x) = 1$

$\frac{\pi}{4}, \frac{5\pi}{4}$ critical





Test

$\left(0, \frac{\pi}{4}\right)$ $\frac{\pi}{6}$ $f'\left(\frac{\pi}{6}\right) = 7\left(\cos\frac{\pi}{6} - \sin\frac{\pi}{6}\right) > 0$ +

$\left(\frac{\pi}{4}, \frac{5\pi}{4}\right)$ π $f'(\pi) = 7(\cos\pi - \sin\pi) < 0$ −

$\left(\frac{5\pi}{4}, 2\pi\right)$ $\frac{3\pi}{2}$ $f'\left(\frac{3\pi}{2}\right) = 7\left(\cos\frac{3\pi}{2} - \sin\left(\frac{3\pi}{2}\right)\right) > 0$ +

$f\left(\frac{\pi}{4}\right) = 7\left(\sin\frac{\pi}{4} + \cos\frac{\pi}{4}\right)$

$= 7\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right)$

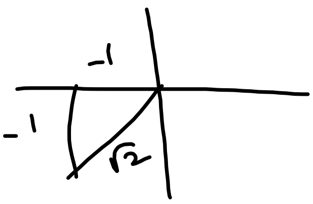
$= \frac{14}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{14\sqrt{2}}{2} = 7\sqrt{2}$

Local max @ $\frac{\pi}{4}$

$f\left(\frac{5\pi}{4}\right) = 7\left(\sin\frac{5\pi}{4} + \cos\frac{5\pi}{4}\right)$

$7\left(-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\right) = -7\sqrt{2}$ @ $x = \frac{5\pi}{4}$

LOCAL MIN.



(c) Find the inflection points. (Order your answers from smallest to largest x, then from smallest to largest y.)

$(x, y) = ($ ✗ $\left(\frac{3\pi}{4}, 0\right)$ $)$ ✓

$(x, y) = ($ ✗ $\left(\frac{7\pi}{4}, 0\right)$ $)$ ✓

$f'(x) = 7(\cos(x) - \sin(x))$

$f''(x) = 7(-\sin(x) - \cos(x))$ set $= 0$

$\sin(x) = -\cos x$

$\tan(x) = -1$

Find the interval on which f is concave up. (Enter your answer using interval notation.)

✗ $\left(\frac{3\pi}{4}, \frac{7\pi}{4}\right)$

$x = \frac{3\pi}{4}, \frac{7\pi}{4}$

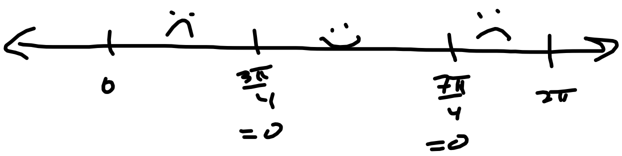
$f\left(\frac{3\pi}{4}\right) = 7\left(\sin\left(\frac{3\pi}{4}\right) + \cos\left(\frac{3\pi}{4}\right)\right)$

Find the interval on which f is concave down. (Enter your answer using interval notation.)

✗ $\left(0, \frac{3\pi}{4}\right), \left(\frac{7\pi}{4}, 2\pi\right)$

$= 7\left(\frac{1}{\sqrt{2}} + \left(-\frac{1}{\sqrt{2}}\right)\right) = 0$

f''



$f''(x) = -7(\sin(x) + \cos(x))$

Test: $\frac{\pi}{2}, \pi, \frac{11\pi}{6}$

$\left(0, \frac{3\pi}{4}\right)$ $\frac{\pi}{2}$ $-7\left(\sin\frac{\pi}{2} + \cos\frac{\pi}{2}\right) = -7 < 0$

$\left(\frac{3\pi}{4}, \frac{7\pi}{4}\right)$ π $-7(\sin\pi + \cos\pi) = -7(0 - 1) = 7 > 0$

$\left(\frac{7\pi}{4}, 2\pi\right)$ $\frac{11\pi}{6}$ $-7\left(\sin\frac{11\pi}{6} + \cos\frac{11\pi}{6}\right) = -7\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}\right) < 0$