$f'(t) = \sec(t)(\sec(t) + \tan(t)), \quad -\frac{\pi}{2} < t < \frac{\pi}{2}, \quad f\left(\frac{\pi}{4}\right) = -5$

$= \sec^2(t) + \sec(t)\tan(t) \Longrightarrow$

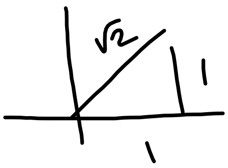
$f(t) = \int \left(\sec^2(t) + \sec(t)\tan(t)\right)dt + C$

$= \boxed{\tan(t) + \sec(t) + C = f(t)}$

$f\left(\frac{\pi}{4}\right) = -5 \Longrightarrow$

$\tan\left(\frac{\pi}{4}\right) + \sec\left(\frac{\pi}{4}\right) + C = -5$

$1 + \sqrt{2} + C = -5 \Longrightarrow$



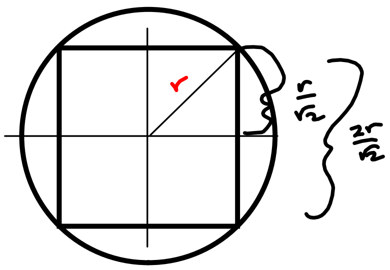
$c = -5 - 1 - \sqrt{2} = \boxed{-6 - \sqrt{2} = c}$

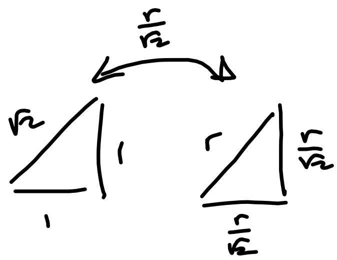
c = "constant of integration."

Find the dimensions of the rectangle of largest area that can be inscribed in a circle of radius r.

width [ ] ✖ $\sqrt{2}r$ units

height [ ] ✖ $\sqrt{2}r$ units





$$\text{Area} \stackrel{?}{=} \left(\frac{2r}{\sqrt{2}}\right)^2 = \frac{4r^2}{2} = 2r^2$$

$$2\left(\frac{r}{\sqrt{2}}\right)\left(\frac{\sqrt{2}}{\sqrt{2}}\right) = \frac{2r\sqrt{2}}{2} = \sqrt{2}\, r$$

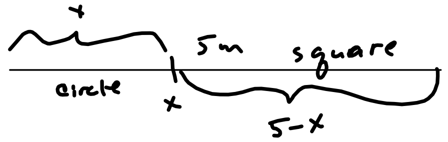
A piece of wire 5 m long is cut into two pieces. One piece is bent into a square and the other is bent into an equilateral triangle.

(a) How much wire should be used for the square in order to maximize the total area?

$\boxed{5}$ m

(b) How much wire should be used for the square in order to minimize the total area?

$\boxed{\dfrac{20\sqrt{3}}{9+4\sqrt{3}}}$ m



Maximize area $= \frac{1}{4\pi}x^2 + \left(\frac{x-5}{4}\right)^2$

Area of circle of perimeter x

$2\pi r = x \longrightarrow$

$r = \frac{x}{2\pi}$ &

Area $= \pi r^2 = \pi\left(\frac{x}{2\pi}\right)^2 = \frac{\pi x^2}{4\pi^2} = \frac{x^2}{4\pi}$

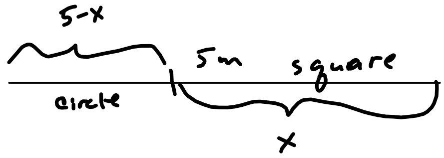
Area of square of perimeter $5-x$

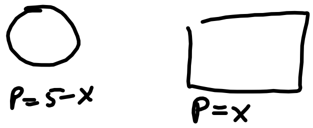
Each side is $\frac{5-x}{4}$ &

Area $= \left(\frac{5-x}{4}\right)^2 = \left(\frac{x-5}{4}\right)^2$

Maximize area $= \frac{1}{4\pi}x^2 + \left(\frac{x-5}{4}\right)^2 = A(x) \longrightarrow$

This is a parabola with a MINIMUM and no maximum!!!





$2\pi r = 5 - x$

$r = \frac{5-x}{2\pi}$

$\pi r^2 = \frac{(5-x)^2}{4\pi^2}\pi = \frac{(5-x)^2}{4\pi}$

$\frac{x}{4}$ = length of a side.

Area = $\left(\frac{x}{4}\right)^2 = \frac{x^2}{16}$

Area of ○ + □ = $\frac{(5-x)^2}{4\pi} + \frac{x^2}{16}$ has No ~~max~~ local Max.

So $0 \leq x \leq 5$. E.V.T.

Cnt's Function DOES achieve its max/min on a closed interval.

Technique? $y' = 0$ No help for a max

$A(0) = \frac{5^2}{4\pi}$

$A(5) = \frac{5^2}{16}$

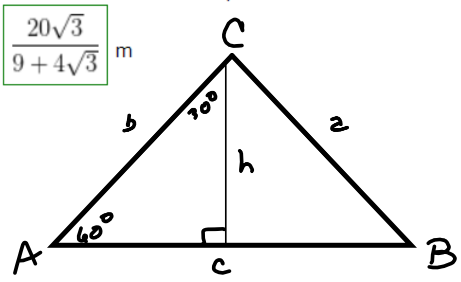
A piece of wire 5 m long is cut into two pieces. One piece is bent into a square and the other is bent into an equilateral triangle.

(a) How much wire should be used for the square in order to maximize the total area?

✗ 5 m

(b) How much wire should be used for the square in order to minimize the total area?

✗ $\frac{20\sqrt{3}}{9+4\sqrt{3}}$ m



$\frac{1}{2}bh = \frac{1}{2}b(b\sin 60^\circ)$

$\text{Area} = \frac{1}{2}bc\sin A$
$= \frac{1}{2}b^2\sin 60^\circ$

x = perimeter of square
$5-x$ = " " triangle

so $\frac{5-x}{3}$ = length of a side.

$\text{Area} = A(x) = \frac{x^2}{16} + \frac{1}{2}\left(\frac{5-x}{3}\right)^2\left(\frac{\sqrt{3}}{2}\right)$

Maximize this on [0, 5]:

The calculus will find us a MINIMUM,

so use E.V.T.

$A'(x)=0$ no help.

check endpoints!

$A(0) = \frac{1}{2}\left(\frac{5}{3}\right)^2\left(\frac{\sqrt{3}}{2}\right) = \frac{25\sqrt{3}}{36}$

$A(5) = \frac{25}{16} > \frac{25\sqrt{3}}{16} \rightarrow$ $\boxed{x=5 \text{ maximizes Area}}$

$\text{Area} = A(x) = \frac{x^2}{16} + \frac{1}{2}\left(\frac{5-x}{3}\right)^2\left(\frac{\sqrt{3}}{2}\right)$

(b) Minimize the total area. Now calculus can help.

$$A'(x) = \frac{x}{8} + 2\left(\frac{1}{2}\right)\left(\frac{5-x}{3}\right)\left(-\frac{1}{3}\right)\left(\frac{\sqrt{3}}{2}\right)$$

$$= \frac{x}{8} - \frac{1}{3}\cdot\frac{\sqrt{3}}{2}\left(\frac{5-x}{3}\right)$$

$$= \frac{x}{8} - \frac{\sqrt{3}}{18}(5-x) = \frac{1}{8}x - \frac{5\sqrt{3}}{18} + \frac{\sqrt{3}}{18}x$$

$$= \left(\frac{1}{8} + \frac{\sqrt{3}}{18}\right)x - \frac{5\sqrt{3}}{18} \underset{=}{\text{SET}} 0 \longrightarrow$$

$$\left(\frac{1}{8} + \frac{\sqrt{3}}{18}\right)x = \frac{5\sqrt{3}}{18} \longrightarrow$$

$$x = \left(\frac{5\sqrt{3}}{18}\right)\frac{1}{\frac{1}{8} + \frac{\sqrt{3}}{18}}$$

$2\cdot2\cdot2 \quad 3\cdot3\cdot2$

$2^3\cdot3^2 = 72 = LCD$

$$= \frac{5\sqrt{3}}{18}\left(\frac{1}{\frac{9}{72} + \frac{4\sqrt{3}}{72}}\right)$$

$\frac{1}{2^3}\cdot\frac{3^2}{3^2} + \frac{\sqrt{3}}{2\cdot3^2}\cdot\frac{2^2}{2^2}$

$$= \frac{5\sqrt{3}}{18}\left(\frac{1}{\frac{9+4\sqrt{3}}{72}}\right)$$

$$= \frac{5\sqrt{3}}{18}\left(\frac{72}{4\sqrt{3}+9}\right) = \boxed{5\sqrt{3}\left(\frac{4}{4\sqrt{3}+9}\right) = x}$$

$$= \boxed{\frac{20\sqrt{3}}{4\sqrt{3}+9} = x}$$