3.8 #6

Consider the following equation.

$3x^4 - 8x^3 + 7 = 0, \quad [2, 3]$

(a) Explain how we know that the given equation must have a root in the given interval.

Let $f(x) = 3x^4 - 8x^3 + 7$. The polynomial f is continuous on [2, 3], $f(2) =$ ____ ✗ -9 < 0, and

$f(3) =$ ____ ✗ 34 > 0, so by the Intermediate Value Theorem, there is a number c in (2, 3) such that

$f(c) =$ ____ ✗ 0 . In other words, the equation $3x^4 - 8x^3 + 7 = 0$ has a root in [2, 3].

(b) Use Newton's method to approximate the root correct to six decimal places.

____ ✗ 2.521041

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C4 =F3

n	xn	f(xn)	f'(xn)	xn - f(xn)/f'(xn)
1	2.5	-0.8125	37.5	2.521666667
2	2.521666667	0.024870263	39.80610539	2.521041882
3	2.521041882	2.10498E-05	39.73873157	2.521041352
4	2.521041352	1.51346E-11	39.73867447	2.521041352
5	2.521041352	0	39.73867447	2.521041352
6	2.521041352	0	39.73867447	2.521041352
7	2.521041352	0	39.73867447	2.521041352
8	2.521041352	0	39.73867447	2.521041352
9	2.521041352	0	39.73867447	2.521041352
10	2.521041352	0	39.73867447	2.521041352
11	2.521041352	0	39.73867447	2.521041352
12				
13				
14				
15				

#9 §3.8

$y = (x-7)^2$

Find closest point to (0,0)

Want $(x, f(x))$ to minimize distance from (0,0):

$$d = \sqrt{(x-0)^2 + (y-0)^2}$$

$$g(x) = d^2 = x^2 + y^2 = x^2 + ((x-7)^2)^2$$ to be minimized

$g'(x) = 0$ is where this happens

$$g(x) = x^2 + x^2 - 14x + 49 = 2x^2 - 14x + 49$$ Nope!

$$= 2\left(x^2 - 7x + \left(\tfrac{7}{2}\right)^2\right) + 49 - \tfrac{49}{2}$$

$$= 2\left(x - \tfrac{7}{2}\right)^2 + \tfrac{49}{2}$$

$(h, k) = \left(\tfrac{7}{2}, \tfrac{49}{2}\right)$ is the vertex

College Algebra Method

Calc I method:

$$g'(x) = 2x + 2(x-7) = 4x - 14 \overset{SET}{=} 0$$

$$4x = 14$$

$$x = \tfrac{14}{4} = \tfrac{7}{2}$$

$$g(x) = x^2 + (x-7)^4$$

To be minimized

$$g'(x) = 2x + 4(x-7)^3 \overset{SET}{=} 0$$

Define $f(x) = g'(x) \overset{SET}{=} 0$ & solve w/ Newton's Method

$$f'(x) = 2 + 12(x-7)^2$$

$$f(x) = 2x + 4(x-7)^3$$