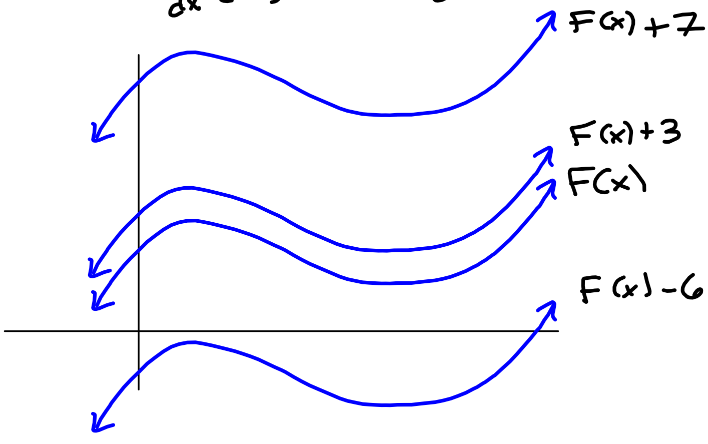
Function	Particular antiderivative	Function	Particular antiderivative
$cf(x)$	$cF(x)$	$\cos x$	$\sin x$
$f(x) + g(x)$	$F(x) + G(x)$	$\sin x$	$-\cos x$
$x^n \ (n \neq -1)$	$\dfrac{x^{n+1}}{n+1}$	$\sec^2 x$	$\tan x$
		$\sec x \tan x$	$\sec x$

If $F'(x) = f(x) \longrightarrow$
$F(x)$ is an antiderivative of f, and so is
any $G(x)$ that differs from $F(x)$ by a constant
We write $\int f(x)\,dx = F(x) + C$ to represent the entire family of anti-derivatives

$$\int x^2\,dx = \frac{x^3}{3} + C, \text{ because}$$

$$\frac{d}{dx}\left[\frac{x^3}{3} + C\right] = \frac{d}{dx}\left[\frac{x^3}{3}\right] + \frac{d}{dx}[C] = 3\left(\frac{x^2}{3}\right) + 0$$



All have same derivative.

The *easiest* anti-derivative is the one with $C = 0$.

Work Derivative tables backwards, sort of.

$F(x)$	$f(x)$	$F(x)$
$\cancel{f(x)}$	$f'(x)$	
$\sin(x)$	$\cos(x)$	
$-\cos(x)$	$+\sin(x)$	$-\cos(x)+C$
$\tan(x)$	$\sec^2(x)$	
$-\csc(x)$	$+\csc(x)\cot(x)$	
$\sec(x)$	$\sec(x)\tan(x)$	
$\cot(x)$	$-\csc^2(x)$	

$$\frac{f'g-fg'}{g^2}$$

$$\frac{d}{dx}\left[\frac{\cos(x)}{\sin(x)}\right]=\frac{-\sin(x)\sin(x)-\cos(x)\cos(x)}{\sin^2(x)}$$

$$=\frac{-1}{\sin^2(x)}=-\csc^2(x)$$

Power Rule

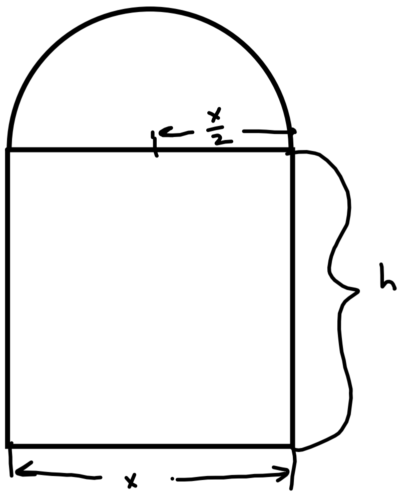
$$\int x^n dx = \frac{x^{n+1}}{n+1} + C$$

Remind me to do "Newton."

A Norman window has the shape of a rectangle surmounted by a semicircle. (Thus the diameter of the semicircle is equal to the width of the rectangle. See the figure below.) If the perimeter of the window is 8 ft, find the value of x so that the greatest possible amount of light is admitted.

3.7 #8

$x =$ ✖ $\boxed{\frac{16}{4+\pi}}$ ft



Perimeter is 8 (Auxiliary Eq'n)
3 sides of rect. plus perimeter of semicircle.

$= x + 2h + 2\pi\left(\frac{x}{2}\right)\left(\frac{1}{2}\right)$

$= x + 2h + \frac{\pi x}{2} = 8 \longrightarrow$

$2h = 8 - x - \frac{\pi}{2}x = 8 - \left(\frac{2+\pi}{2}\right)x$

Maximize
Area = Area of rect. + Area of semicircle

$= xh + \pi\left(\frac{x}{2}\right)^2\left(\frac{1}{2}\right)$

$= x\left(8 - \left(\frac{2+\pi}{2}\right)x \quad\right) + \frac{\pi x^2}{8}$

Derive Falling-Body model, courtesy of Newton.

Let s = height of a falling body with initial height s_0, initial velocity v_0, and acceleration a.

In 'Murrica, $a = -32\ ft/s^2$

$v(t) = \int a(t)\,dt = -\frac{32ft}{s^2} t + C$

$= \int -32\,dt = -32t + C$

$v(0) = C = v_0 \longrightarrow$

$v(t) = -32t + v_0$

$s(t) = \int (-32t + v_0)\,dt = -16t^2 + v_0 t + C$

$s(0) = 0 + 0 + C \longrightarrow C = s_0 \longrightarrow$

$s(t) = -16t^2 + v_0 t + s_0$

$\frac{1}{2}gt^2$