

The graphing problem I messed-up in class, due to wrong sign pattern on f' .

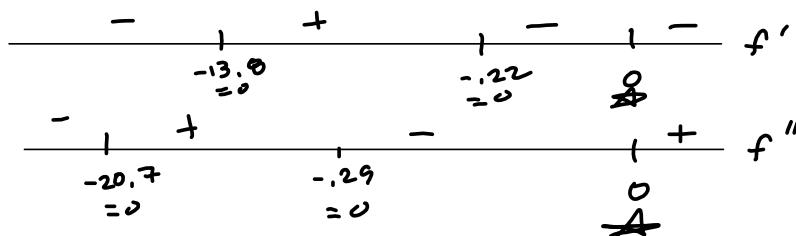
$$f := x \rightarrow 1 + \frac{1}{x} + \frac{7}{x^2} + \frac{1}{x^3}$$

$$f\left(\frac{-21 - \sqrt{417}}{2}\right) \approx 0.9679224558 \approx f(-20.71028893) \text{ IP}$$

$$f(-7 - \sqrt{6}) \approx 0,9639126501 \approx f(-13.78232998) \text{ ~~MAX~~ MIN}$$

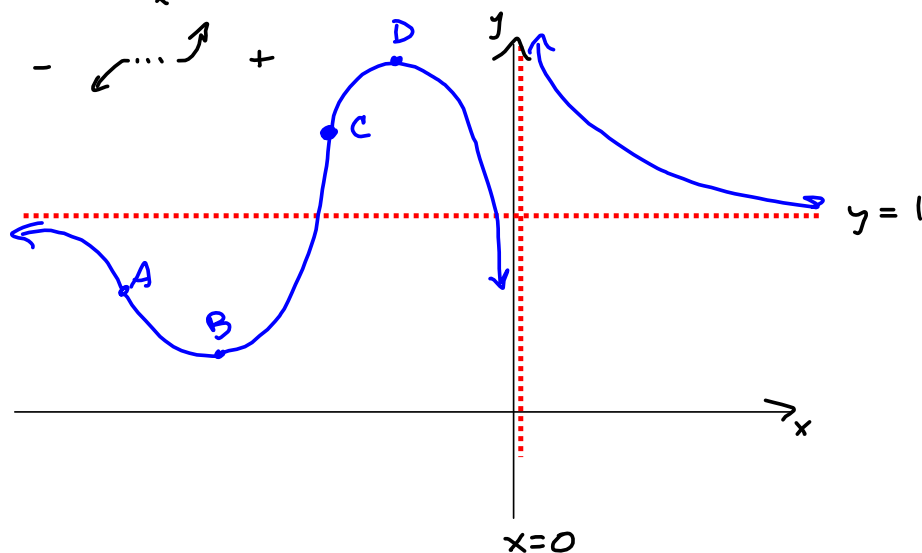
$$f\left(\frac{-21 + \sqrt{417}}{2}\right) \approx 39.82374447 \approx f(-0.28971107) \text{ IP}$$

$$f(-7 - \sqrt{6}) \approx 47.18423548 \approx f(-0.217670017) \text{ ~~MIN~~ MAX}$$



$$f' = - \frac{x^2 + 14x + 3}{x^4} \quad - \swarrow \dots \searrow -$$

$$f'' = \frac{2(x^2 + 21x + 6)}{x^5}$$



3.7 application questions.

Find the points on the ellipse $4x^2 + y^2 = 4$ that are farthest away from the point $(1, 0)$.

$$(x, y) = \left(\boxed{}, \boxed{-\frac{1}{3}, -\frac{4\sqrt{2}}{3}} \right) \text{ (smaller } y\text{-value)}$$

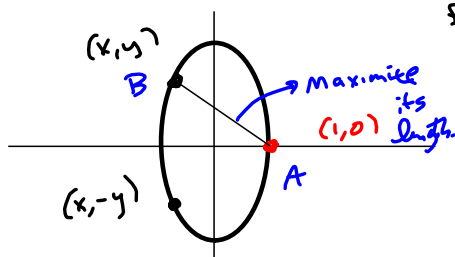
$$(x, y) = \left(\boxed{}, \boxed{-\frac{1}{3}, \frac{4\sqrt{2}}{3}} \right) \text{ (larger } y\text{-value)}$$

$$x^2 + \frac{y^2}{4} = 1$$

$$\Rightarrow y^2 = 4 - 4x^2$$

$$y = \pm \sqrt{4 - 4x^2} = \pm 2\sqrt{1 - x^2}$$

Farthest from $(1, 0)$



Find it for

$$y = 2\sqrt{1 - x^2}$$

& $y = -2\sqrt{1 - x^2}$ is same, only negative y -value.

$$d(A, B) = \sqrt{(x-1)^2 + (y-0)^2} \quad \text{to be maximized}$$

$$f(x, y) = (d(A, B))^2 = (x-1)^2 + y^2 \quad \text{to " "}$$

Now, sub for y :

$$f(x) = (x-1)^2 + (2\sqrt{1-x^2})^2$$

$$= (x-1)^2 + 4(1-x^2)$$

$$= (x-1)^2 + 4 - 4x^2$$

$$\Rightarrow f'(x) = 2(x-1) - 8x$$

$$= 2x - 2 - 8x = -6x - 2 \stackrel{\text{set}}{=} 0$$

$$\Rightarrow -6x = 2$$

$$x = -\frac{2}{6} = -\frac{1}{3}$$

$$\Rightarrow y = 2\sqrt{1 - \left(-\frac{1}{3}\right)^2} = 2\sqrt{1 - \frac{1}{9}}$$

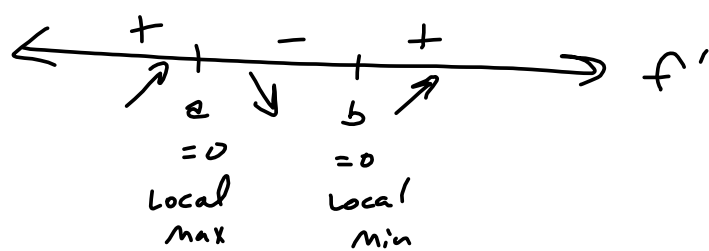
$$= 2\sqrt{\frac{8}{9}} = \frac{2 \cdot 2\sqrt{2}}{3} = \frac{4\sqrt{2}}{3}$$

Solns:

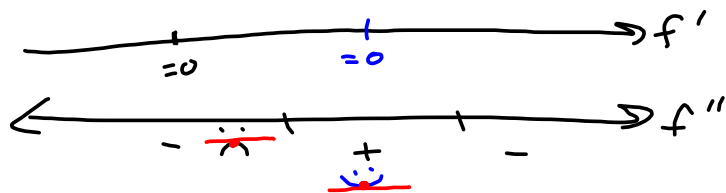
$$\left(-\frac{1}{3}, \frac{4\sqrt{2}}{3}\right)$$

$$\left(-\frac{1}{3}, -\frac{4\sqrt{2}}{3}\right)$$

1st derivative Test



2nd Derivative Test:



$f' = 0$:
 $f'' > 0$ MIN!
 $f'' < 0$ Max.