$\lim\limits_{x \to \infty} f(x) = L$ means that given $\varepsilon > 0$, there is an $N \in \mathbb{N}$ such that for all $x > N$, $|f(x) - L| < \varepsilon$.

A graphing calculator is recommended.

For the limit

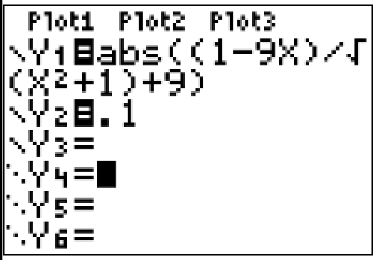
$$\lim_{x \to \infty} f(x) = \lim_{x \to \infty} \frac{1 - 9x}{\sqrt{x^2 + 1}} = -9$$

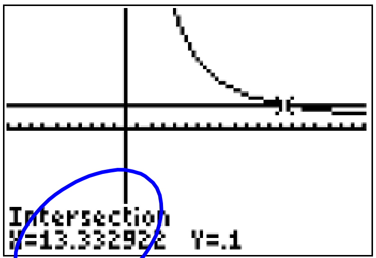
We want to know when $\left| \frac{1-9x}{\sqrt{x^2+1}} - (-9) \right| = .1$

illustrate the definition by finding the smallest integer values of N that correspond to $\varepsilon = 0.1$ and $\varepsilon = 0.05$.

$\varepsilon = 0.1$ $N =$ 14

$\varepsilon = 0.05$ $N =$ 24





$\to N$

We want to find where $|f(x) - (-9)| = .1$

$\boxed{N = 14}$

We found where | f - (-9) | = 0.1. Take the derivative to make sure it'll keep going down and not come back up.

$$\frac{1-9x}{\sqrt{x^2+1}} = f(x) = (1-9x)(x^2+1)^{-\frac{1}{2}}$$

$$\Rightarrow f'(x) = -9(x^2+1)^{-\frac{1}{2}} + (1-9x)\left(-\tfrac{1}{2}(x^2+1)^{-\frac{3}{2}}(2x)\right)$$

$$= \frac{-9}{\sqrt{x^2+1}} + \frac{(1-9x)(2x)}{(-2)(x^2+1)^{3/2}}$$

$$= \frac{-9}{(x^2+1)^{1/2}} \cdot \frac{(x^2+1)}{(x^2+1)} + \frac{(1-9x)(2x)}{-2(x^2+1)^{3/2}}$$

$$= \frac{-9(x^2+1) - (1-9x)x}{(x^2+1)^{3/2}}$$

Need < 0 to guarantee $N = 14$ works

$$= \frac{-9x^2 - 9 - x + 9x^2}{\sim} = \frac{-9-x}{\text{positive}} < 0$$

4 sure!

How large to take *x* so that $\frac{1}{\sqrt{x}} < .0001$

Want $\frac{1}{\sqrt{x}} < .0001 = \frac{1}{10000} \longrightarrow$

$10000 < \sqrt{x}$

$(\sqrt{x})^2 > (10000)^2 \longrightarrow$

$x > 100000000$

Find the limit, if it exists. (If an answer does not exist, enter DNE.)

$$\lim_{x\to\infty}\left(\sqrt{9x^2+x}-3x\right)$$

$$\underbrace{\left(\sqrt{9x^2+x}-3x\right)}_{(A-B)} \xrightarrow{x\to\infty} \infty-\infty \quad \text{No Help} \qquad \underbrace{(A+B)}\quad A^2-B^2$$

$$\left(\frac{\sqrt{9x^2+x}-3x}{1}\right)\left(\frac{\sqrt{9x^2+x}+3x}{\sqrt{9x^2+x}+3x}\right) = \frac{(9x^2+x)-9x^2}{\sqrt{9x^2+x}+3x}$$

$$= \frac{x}{\sqrt{x^2\left(9+\frac{1}{x^2}\right)}+3x} = \frac{x}{\sqrt{x^2}\sqrt{9+\frac{1}{x^2}}+3x} \qquad \frac{x}{|x|\sqrt{9+\frac{1}{x^2}}+3x}$$

$$= \frac{x}{x\left(\sqrt{9+\frac{1}{x^2}}+3\right)} = \frac{1}{\sqrt{9+\frac{1}{x^2}}+3} \xrightarrow{x\to\infty} \frac{1}{\sqrt{9}+3} = \frac{1}{3+3} = \frac{1}{6}$$

Be careful $\sqrt{x^2} \neq x$!

when $x \to -\infty$, $\sqrt{x^2} = |x| = -x$

$$|x| = \begin{cases} x & \text{if } x \geq 0 \quad \left(\lim\limits_{x\to\infty}\right) \\ -x & \text{if } x < 0 \quad \left(\lim\limits_{x\to-\infty}\right) \end{cases}$$

X	Y_1	
10	.16621	
100	.16662	
1000	.16666	
10000	.16667	
100000	.16667	
20	.16644	
15	.16636	

X=20

This says 1/6, not 1/3!!!