Consider the equation below. (If an answer does not exist, enter DNE.)

$f(x) = x^3 - 6x^2 - 15x + 4$

(a) Find the interval on which f is increasing. (Enter your answer using interval notation.)

Find the interval on which f is decreasing. (Enter your answer using interval notation.)

(b) Find the local minimum and maximum values of f. $(-1, 12)$ Local Max $(5, -96)$ Local MiN

(c) Find the inflection point. $(2, -22)$

Find the interval on which f is concave up. (Enter your answer using interval notation.)

$f'(x) = 3x^2 - 15 \overset{SET}{=} 0 \Longrightarrow$ NO

$3x^2 = 15 \Longrightarrow$

$x^2 = \frac{15}{3} = 5$

$x = \pm\sqrt{5}$

$\sqrt{5}\,|\; 1 \quad -6 \quad -15 \quad 4$

$\sqrt{5} \quad -6\sqrt{5}+5 \quad -10\sqrt{5}-30$

$1 \quad -6+\sqrt{5} \quad -10-6\sqrt{5} \quad \boxed{-26-10\sqrt{5}}$

Remainder Theorem:

$f(\sqrt{5}) = -26 - 10\sqrt{5}$

$3x^2 - 12x - 15 = 3(x^2 - 4x - 5)$

$= 3(x-5)(x+1) \overset{SET}{=} 0$

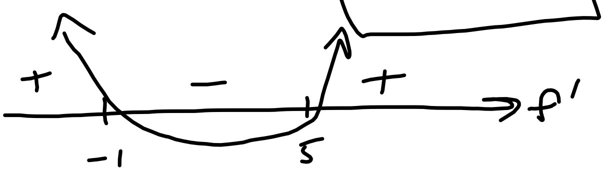
$\Longrightarrow x \in \{-1, 5\}$

$-1\,|\; 1 \quad -6 \quad -15 \quad 4$

$-1 \quad 7 \quad 8$

$1 \quad -7 \quad -8 \quad 12 = f(-1)$

$$
\begin{array}{r|rrrr}
5 & 1 & -6 & -15 & 4 \\
 & & 5 & -5 & -100 \\
\hline
 & 1 & -1 & -20 & \boxed{-96 = f(5)}
\end{array}
$$



$3x^2 - 12x - 15$

+++++ - - - - - - - - ++ +++ f'

−1 5

- - - - - - - ++++++++ f''

2

$$\boxed{\begin{array}{l}\text{concave up: } (2, \infty) \\ \text{concave down } (-\infty, 2)\end{array}}$$

$f''(x) = 6x - 12 \overset{\text{SET}}{=} 0$

$6x = 12$

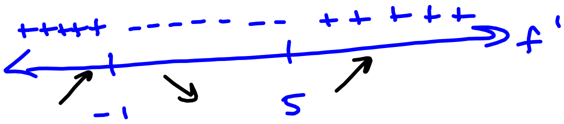
$x = 2$ $\quad f(2):$

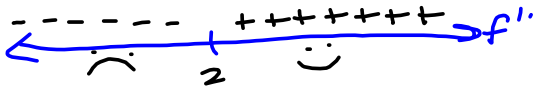
Increasing $(-\infty,-1) \cup (5,\infty)$

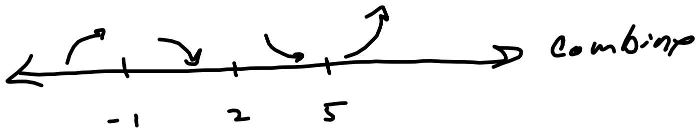
Decreasing $(-1,5)$

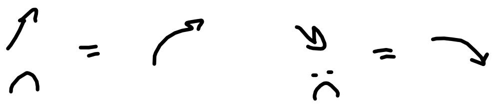
$$\begin{array}{r|rrrr} 2 & 1 & -6 & -15 & 4 \\ & & 2 & -8 & -26 \\ \hline & 1 & -4 & -23 & -22 = f(2) \end{array}$$

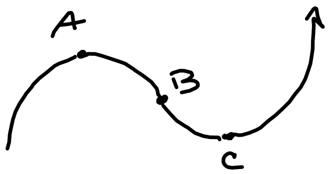
$\boxed{\text{IP: } (2,-22)}$











A = (-1, 12) MAX (Local)

B = (2, -22) INFLECTION

C = (5, -96) MIN (Local)

We do the same thing with a trigonometric polynomial.

Consider the equation below. (If an answer does not exist, enter DNE.)

$f(x) = 6\cos^2(x) - 12\sin(x), \quad 0 \le x \le 2\pi$

(a) Find the interval on which f is increasing. (Enter your answer using interval notation.)

Find the interval on which f is decreasing. (Enter your answer using interval notation.)

(b) Find the local minimum and maximum values of f.

(c) Find the inflection points.

Find the interval on which f is concave up. (Enter your answer using interval notation.)

$f(x) = 6\cos^2(x) - 12\sin(x)$

$\mathcal{D} = \mathbb{R}$

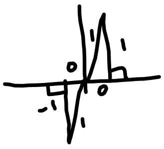
$f'(x) = 12\cos(x)(-\sin(x)) - 12\cos(x) \overset{\text{SET}}{=} 0 \implies$

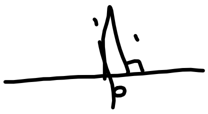
$-12\sin(x)\cos(x) - 12\cos(x) = 0 \implies$

$\sin(x)\cos(x) + \cos(x) = 0$

$\implies \cos(x)(\sin(x) - 1) = 0$

$\implies \cos(x) = 0 \quad$ OR $\quad \sin(x) = 1$





$x = \frac{\pi}{2}, \frac{3\pi}{2}$

$x = \frac{\pi}{2}$

critical #s: $x = \frac{\pi}{2}, \frac{3\pi}{2}$

calculator will only see $\cos^{-1}(0) = \frac{\pi}{2}$

It won't see $\frac{3\pi}{2}$

Computer Algebra/Calc. sees $-\frac{\pi}{2} = \sin^{-1}(-1)$

Restricted sine

$\implies$ restricted range on inverse funcs.

Dec | Inc | Dec

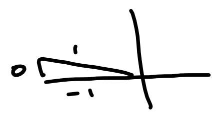
$-$ | $+$ | $-$

$0 \quad \frac{\pi}{2} \quad \frac{3\pi}{2} \quad 2\pi \quad f'$

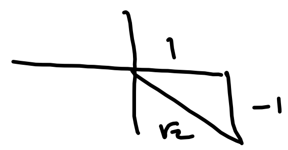
$(0, \frac{\pi}{2})$ $\frac{\pi}{4}$: $6\cos^2(\frac{\pi}{4}) - 12\sin(\frac{\pi}{4})$ $-$

$(\frac{\pi}{2}, \frac{3\pi}{2})$ π $6\cos^2(\pi) - 12\sin(\pi)$ $+$

$(\frac{3\pi}{2}, 2\pi)$ $\frac{7\pi}{4}$ $6\cos^2(\frac{7\pi}{4}) - 12\sin(\frac{7\pi}{4}) = 3 - \frac{12}{\sqrt{2}}$ $-$



$f(x) = 6\cos^2(x) - 12\sin(x)$



f'' & concavity

$f'(x) = 12\cos(x)(-\sin(x)) - 12\cos(x) \overset{SET}{=} 0$

$f'(x) = -12\sin(x)\cos(x) - 12\cos(x) \longrightarrow$

$f''(x) = -12\cos(x)\cos(x) + (-12\sin(x))(-\sin(x)) + 12\sin(x)$

$= -12\cos^2(x) + 12\sin^2(x) + 12\sin(x) \overset{SET}{=} 0$

$\longrightarrow -12(1-\sin^2(x)) + 12\sin^2(x) + 12\sin(x)$

$= -12 + 12\sin^2(x) + 12\sin^2(x) + 12\sin(x)$

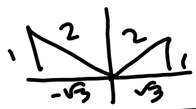
$= 24\sin^2(x) + 12\sin(x) - 12 \overset{\text{SET}}{=} 0$

$\longrightarrow 2\sin^2(x) + \sin(x) - 1$

$(2\sin(x) - 1)(\sin(x) + 1)$

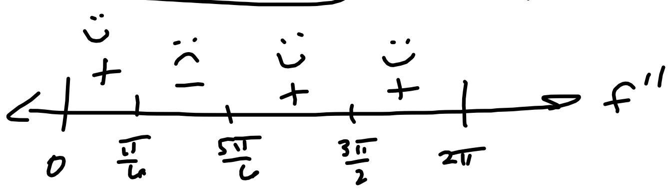
$\sin(x) = \frac{1}{2}$ OR $\sin(x) = -1$

$\boxed{x = \frac{3\pi}{2}}$



$\boxed{x = \frac{\pi}{6}, \frac{5\pi}{6}}$

$\frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$



$\frac{-}{+}$ $\frac{-}{+}$

$\sin(x) + 1 \geq 0$ Always!

f''

$\left(0, \frac{\pi}{6}\right)$ $\frac{\pi}{12}$

$\left(\frac{\pi}{6}, \frac{5\pi}{6}\right)$ $\frac{\pi}{2}$

$\left(\frac{5\pi}{6}, \frac{3\pi}{2}\right)$ π

$\left(\frac{3\pi}{2}, 2\pi\right)$ $\frac{7\pi}{4}$