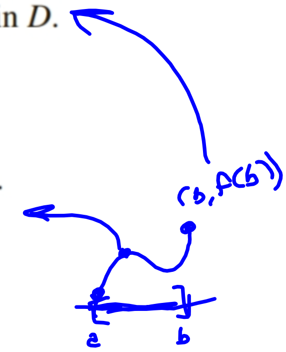
1 Definition Let c be a number in the domain D of a function f. Then $f(c)$ is the

- **absolute maximum** value of f on D if $f(c) \geqslant f(x)$ for all x in D.
- **absolute minimum** value of f on D if $f(c) \leqslant f(x)$ for all x in D.

2 Definition The number $f(c)$ is a

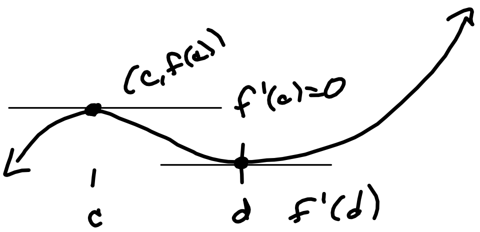
- **local maximum** value of f if $f(c) \geqslant f(x)$ when x is near c.
- **local minimum** value of f if $f(c) \leqslant f(x)$ when x is near c.

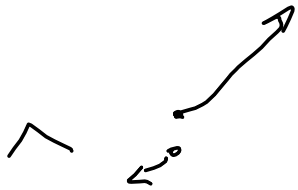


3 The Extreme Value Theorem If f is continuous on a closed interval $[a, b]$, then f attains an absolute maximum value $f(c)$ and an absolute minimum value $f(d)$ at some numbers c and d in $[a, b]$.

4 Fermat's Theorem If f has a local maximum or minimum at c, and if $f'(c)$ exists, then $f'(c) = 0$.

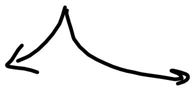
Set $f' = 0$
solve.





6 Definition A **critical number** of a function f is a number c in the domain of f such that either $f'(c) = 0$ or $f'(c)$ does not exist.

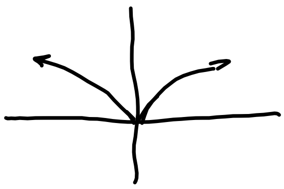
$-x^{2/3}$



$f'(0)\ \nexists$

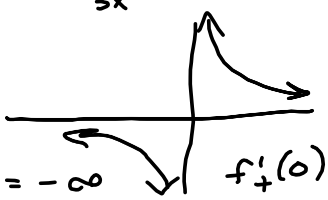
& $f(0)$ is a maximum.

$x^{2/3}$



$$y' = \frac{2}{3}x^{-\frac{1}{3}} = \frac{2}{3x^{1/3}}$$

x^p for $0<p<1$



$f_-'(0) = -\infty$ $\quad$ $f_+'(0) = -\infty$

7 If f has a local maximum or minimum at c, then c is a critical number of f.

Find where $f' = 0$ & where f' DNE
on the domain of f.

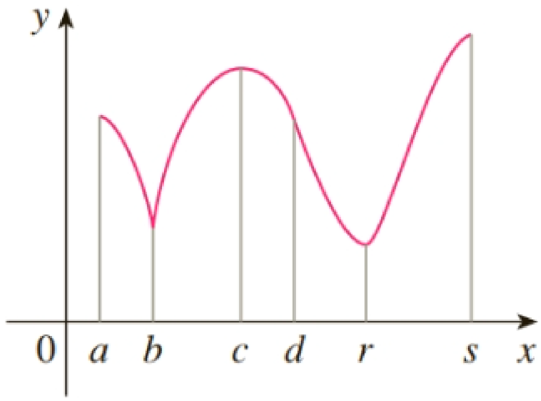
The Closed Interval Method To find the *absolute* maximum and minimum values of a continuous function f on a closed interval $[a, b]$:

1. Find the values of f at the critical numbers of f in (a, b).
2. Find the values of f at the endpoints of the interval.
3. The largest of the values from Steps 1 and 2 is the absolute maximum value; the smallest of these values is the absolute minimum value.

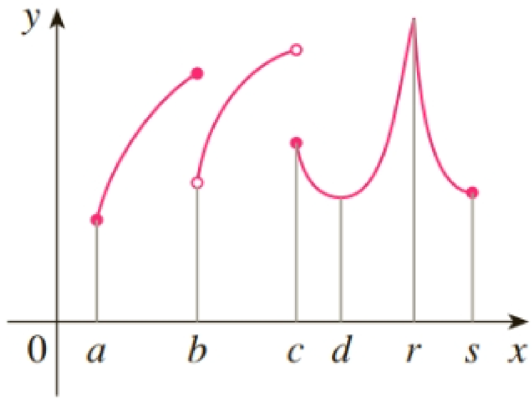
Summary: Check *f* at endpoints of its domain and at all critical values of *f*.

3–4 For each of the numbers a, b, c, d, r, and s, state whether the function whose graph is shown has an absolute maximum or minimum, a local maximum or minimum, or neither a maximum nor a minimum.

3.

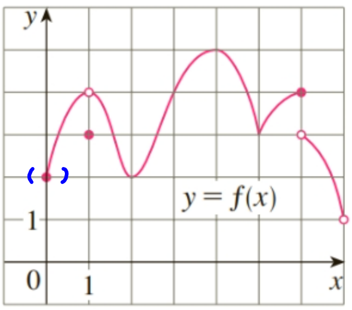


4.



5–6 Use the graph to state the absolute and local maximum and minimum values of the function.

5.



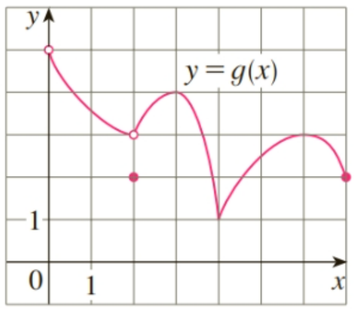
Global Min:
(0,2), (2,2)

Local min:
(2,2), (5,3)
(1,3)

Global Max:
(4,5)

Local Max
(4,5), (7,4)

6.



No Abs Max

Local Max
(3,4), (5,3)

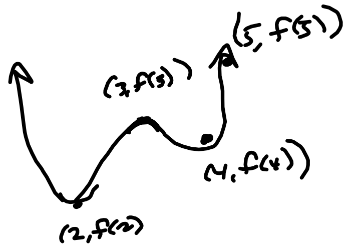
Local min
(2,2) !
(4,1)

Abs min
(4,1)

NOT

7–10 Sketch the graph of a function f that is continuous on $[1, 5]$ and has the given properties.

7. Absolute maximum at 5, absolute minimum at 2, local maximum at 3, local minima at 2 and 4



8. Absolute maximum at 4, absolute minimum at 5, local maximum at 2, local minimum at 3

9. Absolute minimum at 3, absolute maximum at 4, local maximum at 2

10. Absolute maximum at 2, absolute minimum at 5, 4 is a critical number but there is no local maximum or minimum there.

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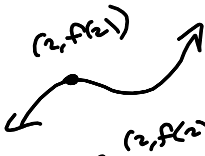
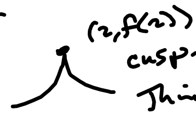
11. (a) Sketch the graph of a function that has a local maximum at 2 and is differentiable at 2.

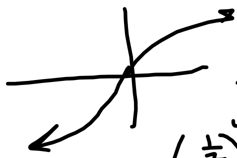


(b) Sketch the graph of a function that has a local maximum at 2 and is continuous but not differentiable at 2.



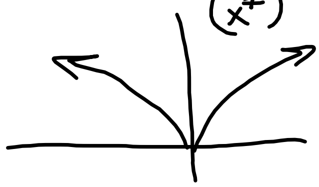
Think $x^{\frac{4}{7}}$

(c) Sketch the graph of a function that has a local maximum at 2 and is not continuous at 2.



$x^{\frac{1}{7}}$

$(x^{\frac{1}{7}})^4$



14. (a) Sketch the graph of a function that has two local maxima, one local minimum, and no absolute minimum.

(b) Sketch the graph of a function that has three local minima, two local maxima, and seven critical numbers.

15–28 Sketch the graph of f by hand and use your sketch to find the absolute and local maximum and minimum values of f. (Use the graphs and transformations of Sections 1.2 and 1.3.)

15. $f(x) = \frac{1}{2}(3x - 1), \quad x \leqslant 3$

16. $f(x) = 2 - \frac{1}{3}x, \quad x \geqslant -2$

17. $f(x) = 1/x, \quad x \geqslant 1$

18. $f(x) = 1/x, \quad 1 < x < 3$

19. $f(x) = \sin x, \quad 0 \leqslant x < \pi/2$

20. $f(x) = \sin x, \quad 0 < x \leqslant \pi/2$

21. $f(x) = \sin x, \quad -\pi/2 \leqslant x \leqslant \pi/2$

22. $f(t) = \cos t, \quad -3\pi/2 \leqslant t \leqslant 3\pi/2$

23. $f(x) = 1 + (x + 1)^2, \quad -2 \leqslant x < 5$

24. $f(x) = |x|$

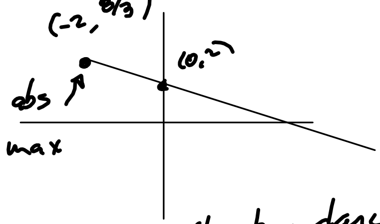
25. $f(x) = 1 - \sqrt{x}$

26. $f(x) = 1 - x^3$

27. $f(x) = \begin{cases} x^2 & \text{if } -1 \leqslant x \leqslant 0 \\ 2 - 3x & \text{if } 0 < x \leqslant 1 \end{cases}$

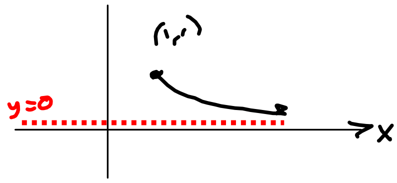
28. $f(x) = \begin{cases} 2x + 1 & \text{if } 0 \leqslant x < 1 \\ 4 - 2x & \text{if } 1 \leqslant x \leqslant 3 \end{cases}$

#16 $f(x) = 2 - \frac{1}{3}x, \quad x \geq -2$

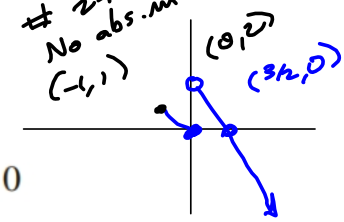


x=2 is its boundary

(17) $f(x) = \frac{1}{x}$



#27
No abs. max



29–42 Find the critical numbers of the function.

29. $f(x) = 4 + \frac{1}{3}x - \frac{1}{2}x^2$

30. $f(x) = x^3 + 6x^2 - 15x$

31. $f(x) = 2x^3 - 3x^2 - 36x$

32. $f(x) = 2x^3 + x^2 + 2x$

33. $g(t) = t^4 + t^3 + t^2 + 1$

34. $g(t) = |3t - 4|$

32 $2x^3 + x^2 + 2x = f(x) \Rightarrow$

$f'(x) = 6x^2 + 2x + 2 \overset{SET}{=} 0$

$\Rightarrow 4x^2 + x + 1 = 0$

$b^2 - 4ac = 1^2 - 4(4)(1) = -15 < 0$

No real zeros

No critical #s

x^p for $0<p<1$ is about the only situation where $f(c)$ exists but $f'(c)$ doesn't

$$x^{\frac{1}{2}} \rightsquigarrow \frac{1}{2x^{1/2}}$$

$$x^{\frac{1}{3}} \rightsquigarrow \frac{1}{3x^{-2/3}}$$

Usually, we just worry about

$$f' = 0$$

$$f' \nexists$$

f' can change signs

$$f'(x) = \frac{(x+2)(x-3)}{\left((x+5)(x-3)\right)^2}$$

45–56 Find the absolute maximum and absolute minimum values of f on the given interval.

45. $f(x) = 12 + 4x - x^2$, $[0, 5]$

46. $f(x) = 5 + 54x - 2x^3$, $[0, 4]$

47. $f(x) = 2x^3 - 3x^2 - 12x + 1$, $[-2, 3]$

48. $f(x) = x^3 - 6x^2 + 5$, $[-3, 5]$

49. $f(x) = 3x^4 - 4x^3 - 12x^2 + 1$, $[-2, 3]$

50. $f(t) = (t^2 - 4)^3$, $[-2, 3]$

51. $f(x) = x + \dfrac{1}{x}$, $[0.2, 4]$

52. $f(x) = \dfrac{x}{x^2 - x + 1}$, $[0, 3]$

53. $f(t) = t - \sqrt[3]{t}$, $[-1, 4]$

#46

$f(x) = -2x^3 + 54x + 5$ on $[0, 4]$

$f(0) = 5 \rightsquigarrow (0, 5)$

4 |	−2	0	54	5
		−8	−32	88
	−2	−8	22	$\boxed{93 = f(4)}$

$\rightsquigarrow (4, 93)$

$f'(x) = -6x^2 + 54$ set $= 0$

$6x^2 = 54$

$x^2 = 9$

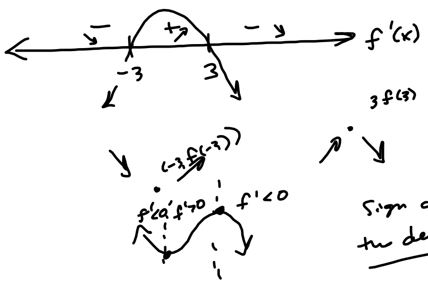
$x = \pm 3$

3 |	−2	0	54	5
		−6	−18	108
	−2	−6	36	$\boxed{113 = f(3)}$

−3 |	−2	0	54	5
		6	−18	−108
	−2	6	36	$\boxed{-103 = f(-3)}$

$(3, 113)$ local & abs

$(-3, -103)$ local & abs.



Sign change of the derivative!

Don't jump to the conclusion that there's a converse to Fermat's Theorem.

Just because the derivative is zero doesn't mean that you're looking at a max or a min:

$f(x) = x^3$

$f'(x) = 3x^2 = 0$ @ $x = 0$, but

$(0, 0)$ isn't a max/min.

It's a "terrace point."