Verify the given linear approximation at $a = 0$. Then determine the values of x for which the linear approximation is accurate to within 0.1. (Enter your answer using interval notation. Round your answers to three decimal places.)

$$\frac{1}{(1+6x)^4} \approx 1 - 24x$$

$x \in$

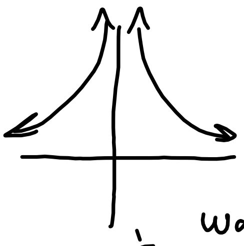
$f(x) = (6x+1)^{-4}$

$f'(x) = -4(6x+1)^{-5}(6)$

$f'(0) = -4(1)^{-5}(6) = -24$

$L(x) = f'(0)(x-0) + f(0)$

$= -24x + 1 = L(x)$ ✓



Want $\left| f(x) - L(x) \right| < 0.1$

$\Rightarrow \left| \frac{1}{(6x+1)^4} - (-24x+1) \right| < .1$

$\Rightarrow \left| \frac{1}{(6x+1)^4} + 24x - 1 \right| < .1$

$-.1 < \frac{1}{(6x+1)^4} + 24x - 1 < .1$

$solve(g(x) - L(x) = 0.1)$

0.01846306269, ~~−0.1784252247 + 0.1060473085 I~~, −0.01511982877, −0.2673261179, ~~−0.1784252247 − 0.1060473085 I~~

$\frac{1}{(6x+1)^4} - (-24x+1)$

Blows up @ $x = -\frac{1}{6}$

$solve(g(x) - L(x) = -0.1)$

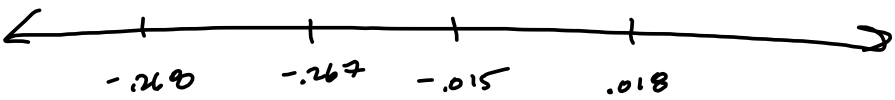
~~−0.001661616377 + 0.01654145020 I~~, ~~−0.1789442488 + 0.1067306374 I~~, −0.2679549364, ~~−0.1789442488 − 0.1067306374 I~~, ~~−0.001661616377 − 0.01654145020 I~~

-.26795, .018463, -.01512, -.2673

-.26795 -.01512 .018463

$g(x) - L(x) < .1$

$g(x) - L(x) - .1 < 0$

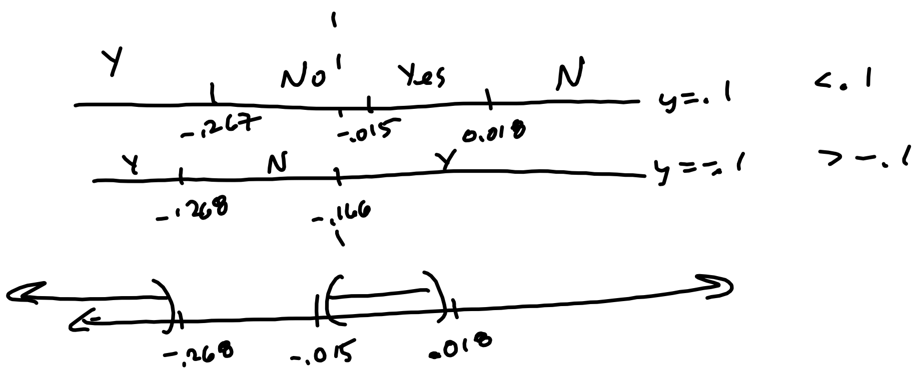


$solve(experiment(x) < .1)$

$$(-\infty, -0.2673261179), (-0.01511982877, 0.01846306269)$$

$solve(experiment(x) > -.1)$

$$(-0.2679549364, -0.1666666667), (-0.1666666667, \infty)$$



Verify the given linear approximation at $a = 0$. Then determine the values of x for which the linear approximation is accurate to within 0.1. (Enter your answer using interval notation. Round your answers to three decimal places.)

$\frac{1}{(1+6x)^4} \approx 1 - 24x$

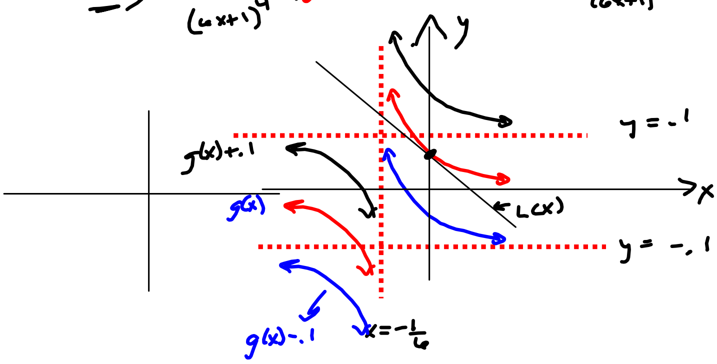
$x \in$

want $\left| \frac{1}{(6x+1)^4} - (1-24x) \right| < .1$

want $-.1 < \frac{1}{(6x+1)^4} + 24x - 1 < .1$

$\Rightarrow -.1 - \frac{1}{(6x+1)^4} < 24x - 1 < .1 - \frac{1}{(6x+1)^4}$

$\Rightarrow \frac{1}{(6x+1)^4} + .1 > -24x + 1 > \frac{1}{(6x+1)^4} - .1$

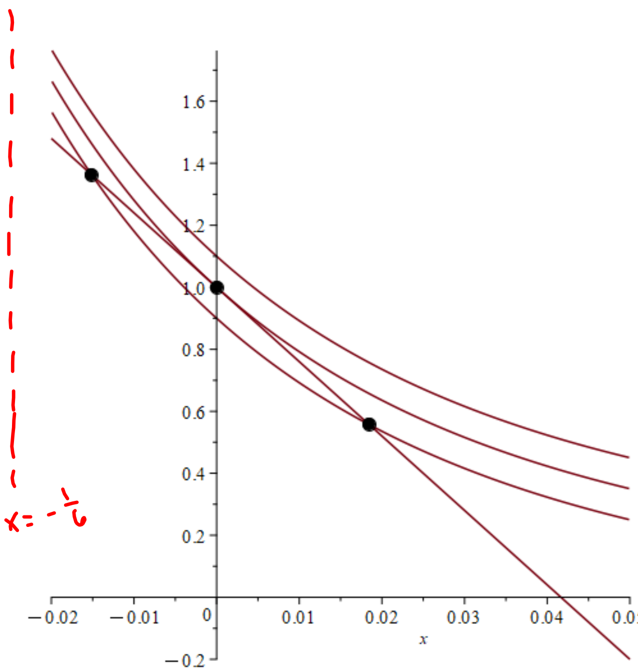


Just looking at this picture, it's evident that the tangent line never hits the g(x) + 0.1. We need the 2 intersections with g(x) - 0.1

$solve(L(x) = g(x) - .1)$

0.01846306269, ~~−0.1784252247 + 0.1060473085 I~~, −0.01511982877, −0.2673261179, ~~−0.1784252247 − 0.1060473085 I~~

WTF ?

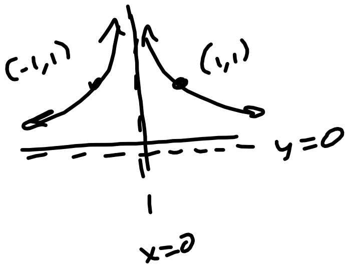


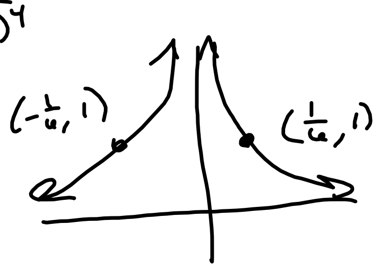
outta bounds

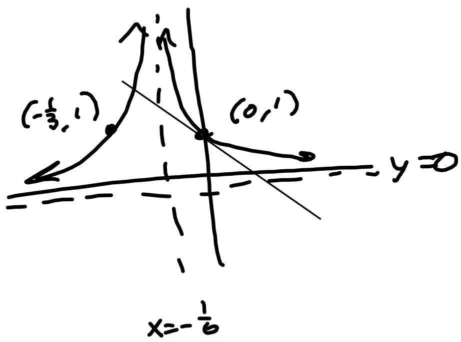
$solve(L(x) = g(x) - .1)$

0.01846306269, ~~−0.1784252247 + 0.1060473085 I~~, −0.01511982877, −0.2673261179, ~~−0.1784252247 − 0.1060473085 I~~

$\frac{1}{x^4} \rightarrow \frac{1}{(6x)^4}$ $\qquad \frac{1}{(6(x+\frac{1}{6}))^4}$







Verify the given linear approximation at $a = 0$. Then determine the values of x for which the linear approximation is accurate to within 0.1. (Enter your answer using interval notation. Round your answers to three decimal places.)

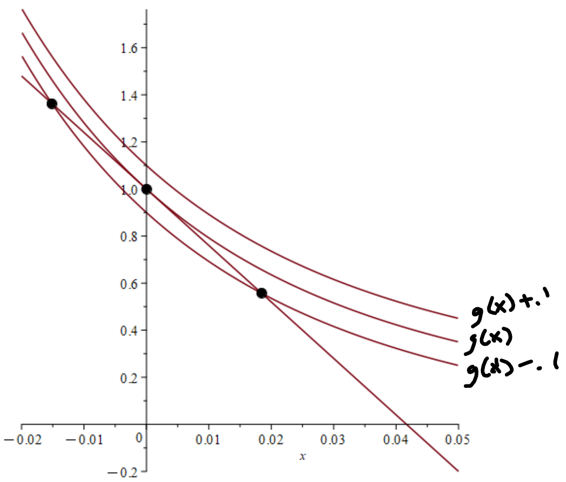
$$\frac{1}{(1+6x)^4} \approx 1 - 24x$$

$x \in \boxed{(-.015,.018)}$

$\text{solve}(L(x) = g(x) - .1)$

$0.01846306269, \cancel{-0.1784252247 + 0.1060473085\,I}, -0.01511982877, -0.2673261179, \cancel{-0.1784252247 - 0.1060473085\,I}$

Left of asymptote



$y = \sin(\sqrt{2}t)$

Find the differential (of y)

$dy = \left(\cos(\sqrt{2}t)\cdot\sqrt{2}\right)dx$

2.9 #9

Compute Δy and dy for the given values of x and $dx = \Delta x$.

$f(x) = y = x^2 - 6x, \quad x = 5, \quad \Delta x = 0.5$

$x_1 = x = 5, \ \Delta x = .5 \longrightarrow$

$x + \Delta x = 5.5 = x_2$

Δy = Net change

$= f(x_2) - f(x_1) = f(5.5) - f(5) = 5.5^2 - 6(5.5) - (5^2 - 6(5))$

$= 30.25 - 33 - 25 + 30 = 60.25 - 58 = \boxed{2.25 = \Delta y}$

$\Delta y \approx dy = f'(x)dx = (2x-6)dx$

$x = 5, dx = .5 \Rightarrow dy = (2(5)-6)(.5) = 4(.5) = \boxed{2 = dy}$

$5.5 = 5 + .5$

$(5.5)^2 = (5 + .5)^2 = 5^2 + 2(5)(.5) + .5^2$

$= 25 + 5 + .25$

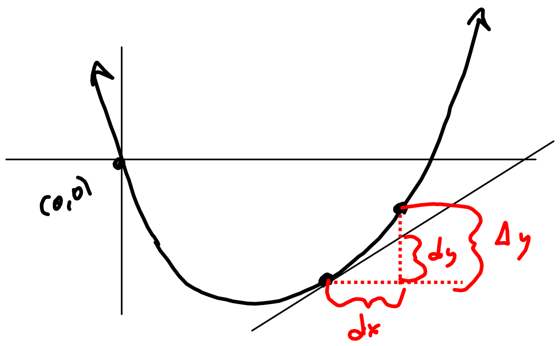
$= 30.25$

$A^2 - B^2$

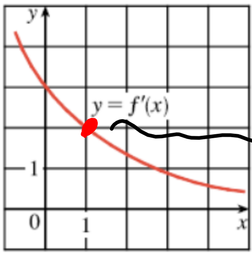
$= (A+B)(A-B)$

#9 Sketch a diagram showing the line segments with lengths dx, dy, and Δy.

$x^2 - 6x$, $dy, \Delta y, \Delta x$



Suppose that the only information we have about a function f is that $f(1) = 8$ and the graph of its *derivative* is as shown.



$(1,2)$

$= (1, f'(1))$

$f(1) = 8$

$L(x) = f'(1)(x-1) + 8$

$= 2(x-1)+8$

$= 2x-2+8$

$= \boxed{2x+6 = L(x)}$

(a) Use a linear approximation to estimate $f(0.99)$ and $f(1.01)$.

$f(0.99) \approx$

$f(1.01) \approx$

$L(.99) = 2(.99)+6$

$= 1.98+6 = \boxed{7.98 \approx f(.99)}$

$L(1.01) = 2(1.01)+6$

$= 2.02+6 = \boxed{8.02 \approx f(1.01)}$

The edge of a cube was found to be 15 cm with a possible error in measurement of 0.1 cm. Use differentials to estimate the maximum possible error, relative error, and percentage error in computing the volume of the cube and the surface area of the cube. (Round your answers to four decimal places.)

(a) the volume of the cube

maximum possible error		cm^3
relative error		
percentage error		%

(b) the surface area of the cube

maximum possible error		cm^2
relative error		
percentage error		%

$V = x^3$, $S = 6x^2$

$dV = 3x^2 dx$

$x = 15$, $dx = \pm .1$

$dV = 3(15)^2(\pm .1)$

$= 3(225)(\pm .1)$

$= (675)(\pm .1)$

$= \pm 67.5 \text{ cm}^3$

225 $.5$

$.5^3$

Relative error $= \dfrac{\text{Error}}{\text{Functional value}} = \dfrac{\Delta V}{V} \approx \dfrac{dV}{V}$

$= \dfrac{\pm 67.5}{3375}$

Percent Error $= (\text{Relative Error})(100\%)$