2.8 - Related Rates

14. If a snowball melts so that its surface area decreases at a rate of 1 cm^2/min, find the rate at which the diameter decreases when the diameter is 10 cm.

Surface area of a sphere of radius *r*: $S = 4\pi r^2 = S(r)$

$= 4\pi\left(\frac{1}{2}D\right)^2 = \pi D^2$ = surface area of a sphere (in cm^2) as a function of D = its diameter (in cm)

WANT: $\left.\frac{dD}{dt}\right|_{D=10}$ Given $\frac{dS}{dt} = -1 \frac{cm^2}{min}$

$\frac{dS}{dD} = 2\pi D$

$\frac{dS}{dt} = \frac{dS}{dD} \cdot \frac{dD}{dt}$

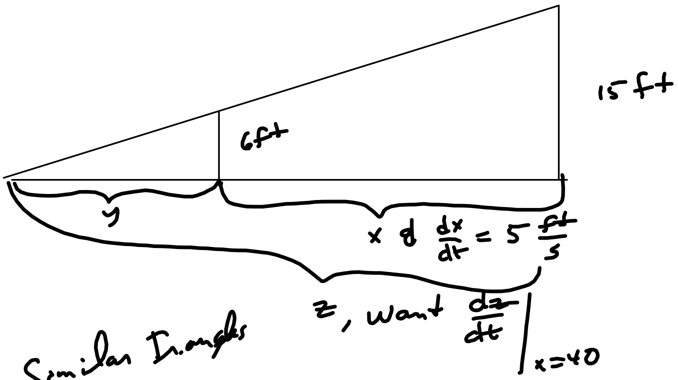
$-1 = 2\pi D \frac{dD}{dt} \longrightarrow \frac{cm^2}{s} = (cm)\frac{cm}{s}$

$D = 10 \Rightarrow$

$-1 = 20\pi \cdot \frac{dD}{dt}$ $\frac{cm}{s} = \frac{cm}{s}$

$\Rightarrow \boxed{\left.\frac{dD}{dt}\right|_{D=10} = -\frac{1}{20\pi} \frac{cm}{s}}$

15. A street light is mounted at the top of a 15-ft-tall pole. A man 6 ft tall walks away from the pole with a speed of 5 ft/s along a straight path. How fast is the tip of his shadow moving when he is 40 ft from the pole?



Similar Triangles

$$\frac{15}{z} = \frac{6}{y}$$

$$\frac{15}{x+y} = \frac{6}{y}$$

$$15y = 6x + 6y$$

$$-6y = \quad -6y$$

$$9y = 6x$$

$$y = \frac{6}{9}x = \boxed{\frac{2}{3}x = y}$$ Auxiliary Eq'n

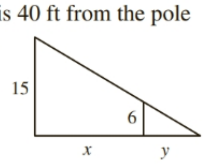
Now

$$z = x + y \longrightarrow$$

$$\frac{dz}{dt} = \frac{d}{dt}\left[x + \frac{2}{3}x\right] = \frac{d}{dt}\left[\frac{5}{3}x\right] \longrightarrow$$

$$\left.\frac{dz}{dt}\right|_{x=40} = \left.\frac{5}{3}\cdot\frac{dx}{dt}\right|_{x=40} = \left.\frac{5}{3}(5) = \boxed{\frac{25}{3}\,\frac{ft}{s}}\right|_{x=40} = \frac{25}{3}\,\frac{ft}{s}$$

15. (a) The height of the pole (15 ft), the height of the man (6 ft), and the speed of the man (5 ft/s)
(b) The rate at which the tip of the man's shadow is moving when he is 40 ft from the pole
(c)



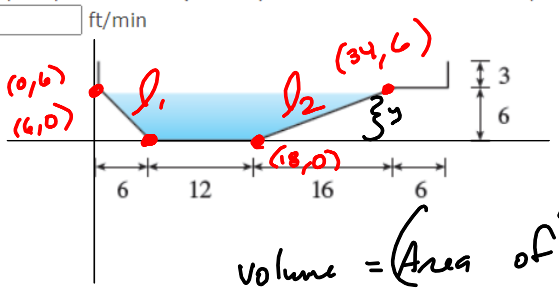
(d) $\frac{15}{6} = \frac{x+y}{y}$ (e) $\frac{25}{3}$ ft/s

A swimming pool is 20 ft wide, 40 ft long, 3 ft deep at the shallow end, and 9 ft deep at is deepest point. A cross-section is shown in the figure. If the pool is being filled at a rate of 0.8 ft^3/min, how fast is the water level rising when the depth at the deepest point is 5 ft? (Round your answer to five decimal places.)

[ ] ft/min

y = depth of water (in ft)

2.8 #12



volume = (Area of the trapezoid as a func. of y)(20 ft)

$= \left(\frac{1}{2}(b_1+b_2)y\right)(20)$

$= \frac{1}{2}($12 + hor. distance from to$)(20y)$

l_1: $m = \frac{6-0}{0-6} = -1$

$y = -1x+6 \rightarrow y-6 = -x \rightarrow \boxed{x = 6-y} = x_1$

l_2: $m = \frac{6-0}{34-18} = \frac{6}{16} = \frac{3}{8}$

$y = \frac{3}{8}(x-18)+0 =$

$= \frac{3}{8}x - \frac{3(18)}{8} = \frac{3}{8}x - \frac{27}{4} = y$

$\frac{3}{8}x = y + \frac{27}{4}$

$x = \frac{8}{3}\left(y+\frac{27}{4}\right) = \frac{8}{3}y + \frac{8(27)}{(3)(4)} = \boxed{\frac{8}{3}y + 18 = x_2}$

For a given y:

(Distance between l_1 & l_2)

$= x_2 - x_1 = \frac{8}{3}y + 18 - (6-y) = \frac{8}{3}y + y - 6$

$= \frac{11}{3}y - 6 = b_2(y)$

Now, $V(y) = \frac{1}{2}(b_1+b_2)y$

$V(y) = \frac{1}{2}\left(12 + \frac{11}{3}y - 6\right)y = \left(3 + \frac{11}{6}y\right)y = \frac{11}{6}y^2 + 3y$

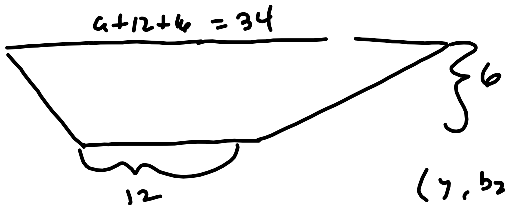
want $\left.\frac{dy}{dt}\right|_{y=5}$. we have $\left.\frac{dV}{dt}\right|_{y=5} = \left.\left(\frac{11}{3}yy' + 3y'\right)\right|_{y=5} = \left.\left(\frac{11}{3}y+3\right)y'\right|_{y=5}$

$.8 \frac{ft^3}{min} = \left(\frac{11}{3}(5)+3\right)\left.\frac{dy}{dt}\right|_{y=5}$

$\rightarrow .8 = \left(\frac{55}{3}+\frac{9}{3}\right)y' = \frac{64}{3}y' \rightarrow$

$\left(\frac{8}{10}\right)\left(\frac{3}{64}\right) = \frac{3}{80}$ ft/min

Easier way to get that 2nd base of the trapezoid:



$y=6: \quad b_2 = 34 \rightsquigarrow (6, 34)$

$y=0: \quad b_2 = 12 \rightsquigarrow (0, 12)$

(y, b_2)

$m = \frac{34-12}{6} = \frac{22}{6} = \frac{11}{3}$

$b_2 = \frac{11}{3}(y-0) + 12$

$$V = \frac{1}{2}(b_1+b_2)y$$

$$= \frac{1}{2}\left(12 + \left(\frac{11}{3}y + 12\right)\right)(20y)$$

$$= \frac{1}{2}\left(\frac{11}{3}y + 24\right)(20y) = \left(\frac{11}{6}y + 12\right)(20y)$$

$$V = \left(\frac{220}{6}y + 240\right)y = \frac{110}{3}y^2 + 240y$$

$$\Rightarrow \frac{dV}{dt} = \frac{220}{3}y\,y' + 240y'$$

$$0.8 = \frac{220}{3}yy' + 240y'$$

(a) $y=5$:

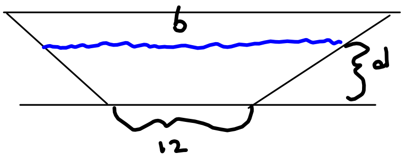
$$\frac{4}{5} = \frac{220}{3}(5)y' + 240y'$$

$$= \frac{1100}{3}y' + 240y'$$

$$\frac{4}{5} = \left(\frac{1100}{3} + 240\right)y'$$

$$= \frac{1100+720}{3}y' = \frac{1820}{3}y'$$

$$y' = \frac{4}{5}\left(\frac{3}{1820}\right) \approx .00132$$


6
d
12