(a) The curve $y = |x|/\sqrt{5 - x^2}$ is called a *bullet-nose curve*. Find an equation of the tangent line to this curve at the point (2, 2). **2.5 #14**

(b) Illustrate part (a) by graphing the curve and the tangent line on the same screen.

$$y = \frac{|x|}{\sqrt{5-x^2}} = \begin{cases} \dfrac{x}{\sqrt{5-x^2}} & \text{if } x \geq 0 \\ \dfrac{-x}{\sqrt{5-x^2}} & \text{if } x > 0 \end{cases}$$

We want:

$$y = f'(2)(x-2) + f(2) = L(x) = \text{Tangent Line / Linearization}$$

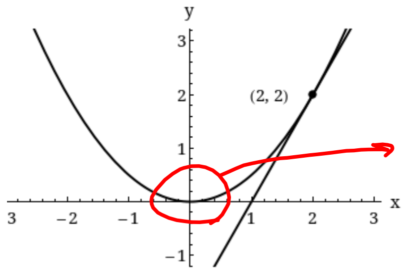
@ $x=2$, we're in the $\dfrac{x}{\sqrt{5-x^2}} = x(5-x^2)^{-\frac{1}{2}} = f(x)$

$$\Rightarrow f'(x) = 1(5-x^2)^{-\frac{1}{2}} + x\left(-\tfrac{1}{2}(5-x^2)^{-\frac{3}{2}}(-2x)\right)$$

$$\Rightarrow f'(2) = (5-(2)^2)^{-\frac{1}{2}} + 2\left(-\tfrac{1}{2}(5-2^2)^{-3/2}(-2(2))\right)$$

$$= 1^{-\frac{1}{2}} + 2\left(-\tfrac{1}{2}(1)^{-3/2}(-4)\right) \qquad 1 + 2\left(-\tfrac{1}{2}\right)(-4) = 1+4 = 5 = m_{tan} = f'(2)$$

For its graph, even with a grapher, it's not immediately obvious.

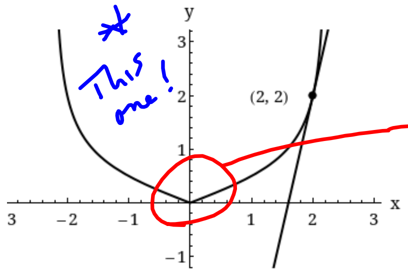


Smooth

$$f'(x) = (5-x^2)^{-\frac{1}{2}} - x^2(5-x^2)^{-\frac{3}{2}}$$

$$= \frac{1}{\sqrt{5-x^2}} \cdot \frac{5-x^2}{5-x^2} - \frac{x^2}{\sqrt{5-x^2}^3}$$

$$= \frac{5-x^2-x^2}{(5-x^2)^{3/2}} = \frac{5-2x^2}{(5-x^2)^{3/2}}$$



* This one!

Seems to come to a point.

$$f'(x) = \begin{cases} \dfrac{5-2x^2}{(5-x^2)^{3/2}} & \text{if } x > 0 \\ \dfrac{2x^2-5}{(5-x^2)^{3/2}} & \text{if } x < 0 \end{cases}$$

What about @ $x=0$?

$$\lim_{x\to 0^-}\left[f'(x)\right] = \lim_{x\to 0^-} \frac{2x^2-5}{(5-x^2)^{3/2}} = f'_-(0) = \frac{-5}{5^{3/2}}$$

$$= \frac{-5}{5^{3/2}}$$

$$\lim_{x\to 0^+}\left[f'(x)\right] = f'_+(0) = \frac{5}{5^{3/2}} \neq \frac{-5}{5^{3/2}} \Rightarrow$$

$f'(0)$ $\nexists$, so it's pointy @ $x=0$, not smooth

2.5

#20

A Cepheid variable star is a star whose brightness alternately increases and decreases. For a certain star, the interval between times of maximum brightness is 5.8 days. The average brightness of this star is 3.0 and its brightness changes by ±0.45. In view of these data, the brightness of the star at time t, where t is measured in days, has been modeled by the function

$$B(t) = 3.0 + 0.45\sin\left(\frac{2\pi t}{5.8}\right).$$

(a) Find the rate of change of the brightness after t days.

$\frac{dB}{dt} =$ ☐ Find $B'(t)$

(b) Find, correct to two decimal places, the rate of increase after five days.

$\frac{dB}{dt} =$ ☐ Find $B'(5)$

$$\text{(a) } B'(t) = .45\left(\cos\left(\frac{2\pi t}{5.8}\right)\right)\cdot\frac{2\pi}{5.8} = \frac{9\pi}{58}\cos\left(\frac{2\pi t}{5.8}\right)$$

$$\frac{.45(2\pi)}{5.8} = \frac{.45\pi}{2.9} = \frac{4.5\pi}{29} = \frac{45\pi}{290} = \frac{9\pi}{58}$$

$$B'(5) = .45\cos\left(\frac{10\pi}{5.8}\right)\cdot\frac{2\pi}{5.8}$$

$$= \frac{9\pi}{58}\cos\left(\frac{10\pi}{5.8}\right)$$

0.3155933791

Two curves are **orthogonal** if their tangent lines are perpendicular at each point of intersection. Are the given families of curves **orthogonal trajectories** of each other? That is, is every curve in one family orthogonal to every curve in the other family?

2.6 $y = cx^2, \; x^2 + 2y^2 = k$

#16

- ◯ Yes, the given families of curves are orthogonal trajectories.
- ◯ No, the given families of curves are not orthogonal trajectories.

Sketch both families of curves on the same axes.

I'm not getting negative reciprocals for the slopes.

$y = cx^2 \Longrightarrow y' = 2cx$

$x^2 + 2y^2 = k$

$2x + 4yy' = 0$

$y' = -\frac{2x}{4y} = -\frac{x}{2y} = \pm\frac{-x}{\sqrt{\frac{k-x^2}{2}}}$ No

$2y^2 = k - x^2$

$y^2 = \frac{k-x^2}{2}$

$y = \pm\sqrt{\frac{k-x^2}{2}}$

The WebAssign "Watch It" is much better than my treatment.