Section 2.5 Chain Rule

$h(x) = f(g(x)) \Longrightarrow$

$h'(x) = f'(g(x)) \cdot g'(x)$

$\frac{dh}{dx} = \frac{df}{dg} \cdot \frac{dg}{dx}$

$\boxed{E}$ $\frac{d}{dx}\left[\sin(x^3+2x^2)\right] = \cos(x^3+2x^2)\cdot(3x^2+4x)$

$= (3x^2+4x)\cos(x^3+2x^2)$

Production of cars (f) as a function of the amount of raw materials (g)

Raw materials as a function of the # of barges available $\to x$

$f(g) = 20g^2$

$g(x) = 10x+7$

Then $h(x) = f(g(x)) = 20\,g(x)^2 = 20(10x+7)^2$

$h'(x) = \underbrace{40(10x+7)}_{\frac{df}{d(10x+7)}} \cdot \underbrace{10}_{\frac{d(10x+7)}{dx}} = 400(10x+7) = 4000x + 2800$

$h(x) = 20(10x+7)^2 = 20(100x^2+140x+49)$

$\Longrightarrow h'(x) = 20(200x+140) = 4000x + 2800$

$h(x) = (\sin(x) - 5x)^{15} \Longrightarrow$

$$h'(x) = \frac{d\,\text{outside}}{d\,\text{inside}} \cdot \frac{d \cdot \text{inside}}{dx} = \boxed{15(\sin(x) - 5x)^{14} \cdot (\cos(x) - 5)}$$

2.3 #24

$f(x) = x^3 + 3x^2 + x + 5$ where's it horizontal?

$f'(x) = 3x^2 + 6x + 1 \overset{\text{SET}}{=} 0$

$f'(x) = 3x^2 + 6x + 1 = 3(x^2 + 2x \quad) + 1$

$= 3(x^2 + 2x + 1^2) - 3 + 1 = 3(x+1)^2 - 2 \overset{\text{SET}}{=} 0$

$\Longrightarrow 3(x+1)^2 = 2$

$\Longrightarrow (x+1)^2 = \frac{2}{3}$

$\Longrightarrow \sqrt{(x+1)^2} = \sqrt{\frac{2}{3}}$

$\Longrightarrow |x+1| = \sqrt{\frac{2}{3}}$

$\Longrightarrow x+1 = \pm\sqrt{\frac{2}{3}}$

$\Longrightarrow x = -1 \pm \sqrt{\frac{2}{3}} = -1 \pm \sqrt{\frac{2 \cdot 3}{3 \cdot 3}}$

$= -1 \pm \frac{\sqrt{6}}{\sqrt{3^2}} = -1 \pm \frac{\sqrt{6}}{3}$ ✓

$b^2 - 4ac = 6^2 - 4(3)(1) = 36 - 12 = 24$

$\sqrt{24} = 2\sqrt{6} \Longrightarrow$

$$x = \frac{-6 \pm 2\sqrt{6}}{2(3)} = \frac{2(-3 \pm \sqrt{6})}{2(3)}$$

$$= \frac{-3 \pm \sqrt{6}}{3} = -1 \pm \frac{\sqrt{6}}{3}$$

$2 | 24$
$2 | 12$
$2 | 6$
3

$Ax + By = C \Rightarrow m = -\frac{A}{B}$

$5x - 3y = 7 \Rightarrow m = \frac{-5}{-3} = \frac{5}{3}$

$By = -Ax + C$

$y = \frac{-A}{B}x + \frac{C}{B}$

FACT: If $y = m_1 x + b_1$ is perpendicular (normal) to $y = m_2 x + b_2$,

then $m_2 = -\frac{1}{m_1}$, i.e. $m_1 = -\frac{1}{m_2}$

Find an equation of the normal line to the curve $y = \sqrt{x}$ that is parallel to the line $2x + y = 1$.

$m_1 = -\frac{A}{B} = -2$

$y = \sqrt{x} \Rightarrow y' = \frac{1}{2}x^{-\frac{1}{2}} = m_x$

we want to be $\perp$ to m_x & $\parallel$ to m_1

$m_x = \frac{1}{2}x^{-\frac{1}{2}}$. we want $\perp$ to that:

$m_{x_\perp} = \frac{-1}{\frac{1}{2}x^{-\frac{1}{2}}} = -2x^{\frac{1}{2}} = -2\sqrt{x} \overset{\text{SET}}{=} -2$

$\Rightarrow \sqrt{x} = 1 \Rightarrow$

$x = 1^2 = 1 = x \rightsquigarrow (1, f(1)) = (x_1, y_1)$

$y_1 = f(1) = \sqrt{1} = 1$

Line thru $(1,1)$ with slope -2

$\boxed{y = -2(x-1) + 1}$

$= -2x + 2 + 1$

$= -2x + 3$

Recall: f, g functions

$\mathcal{D}(f \pm g) = \mathcal{D}(f) \cap \mathcal{D}(g)$

intersect "and"

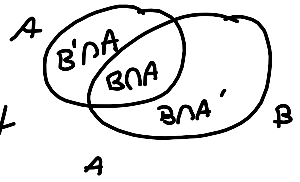
$= \{x \mid x \in \mathcal{D}(f) \text{ and } x \in \mathcal{D}(g)\}$

$= \mathcal{D}(fg)$

$A \cap B = A$ intersect B

$= \{x \mid x \in A \text{ and } x \in B\}$

B' = everything outside of B = Complement of B.



$A \cup B = A$ union B

OR $= \{x \mid x \in A \text{ OR } x \in B\}$

$\mathcal{D}\left(\frac{f}{g}\right) = \mathcal{D}(f) \cap \mathcal{D}(g) \cap \{x \mid g(x) \neq 0\}$

$= \{x \mid x \in \mathcal{D}(f) \text{ and } x \in \mathcal{D}(g) \text{ and } g(x) \neq 0\}$

Domain of the Composition $f \circ g$.

$(f \circ g)(x) = f(g(x))$

$\mathcal{D}(f \circ g) = \{x \mid x \in \mathcal{D}(g) \text{ and } g(x) \in \mathcal{D}(f)\}$

$\mathcal{D}(\sqrt{10 - \sqrt{x}})$

Need $x \geq 0$ for $g(x) = 10 - \sqrt{x}$ b/c $\sqrt{x}$

Need $10 - \sqrt{x} \geq 0$ for $f(g)$

$f(x) = \sqrt{x}$ outer

$f(g) = \sqrt{g}$

$g(x) = 10 - \sqrt{x}$

$\mathcal{D}(g) = [0, \infty) = \{x \mid x \geq 0\}$

$\mathcal{D}(f(g)) = ?$

Need $g \geq 0$

$10 - \sqrt{x} \geq 0$

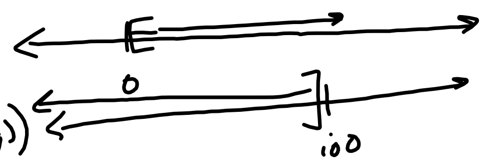
$10 \geq \sqrt{x}$

$\sqrt{x} \leq 10$

$\Rightarrow x \leq 100$!

$\mathcal{D}(g)$ $\cap$ = AND

$\mathcal{D}(f(g))$



$= [0, 100]$