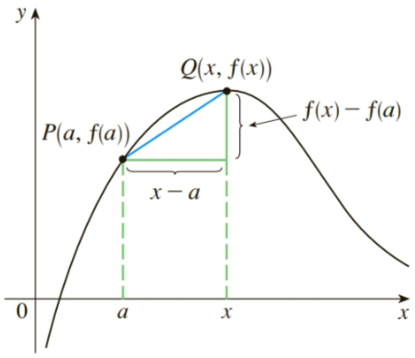
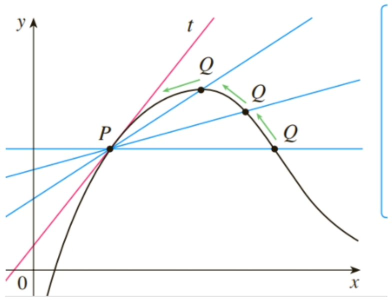
Section 2.1 Derivatives and Rates of Change



Find the tangent line to the Curve C at $x = a$

$$m_{PQ} = \frac{f(x) - f(a)}{x - a}$$ is the slope of the secant line, or average slope of the curve between x and a.



Take x closer and closer to a to get a better estimate of the steepness at $(a, f(a))$

1 Definition The **tangent line** to the curve $y = f(x)$ at the point $P(a, f(a))$ is the line through P with slope

$$m = \lim_{x \to a} \frac{f(x) - f(a)}{x - a} = f'(a)$$

provided that this limit exists.

Then the equation of the tangent line uses that slope m

Book : $y - f(a) = m(x - a)$

Teacher : $y = m(x - a) + f(a)$

m is that limit, call it $f'(a)$, so

$$y = f'(a)(x - a) + f(a)$$
$$= f(a) + f'(a)(x - a)$$

$$y = y_1 + m(x - x_1)$$
$$= m(x - x_1) + y_1$$

Smooth curves are locally linear.

The ant doesn't know the Earth is round.

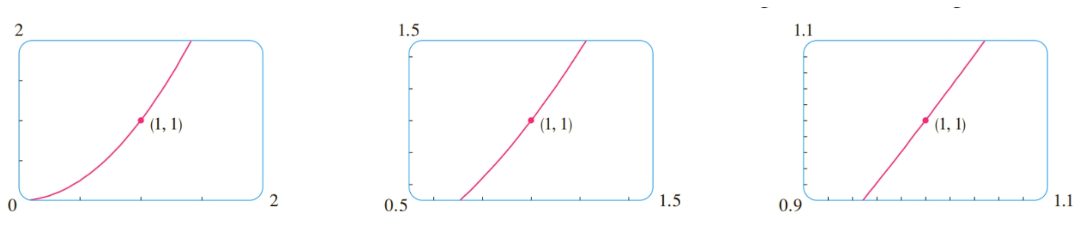


FIGURE 2 Zooming in toward the point (1, 1) on the parabola $y = x^2$

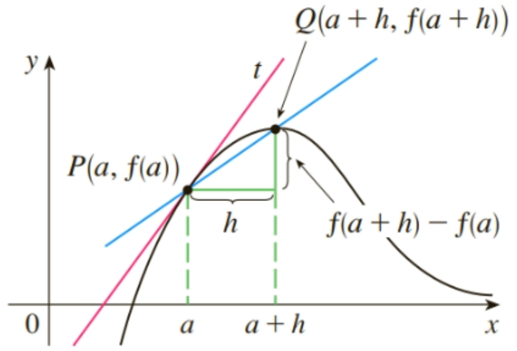


FIGURE 3

Alternate formulation we use quite a bit:

$x = a + \text{some \#} = a + h.$

Then $\lim_{x \to a} \frac{f(x) - f(a)}{x - a} = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$

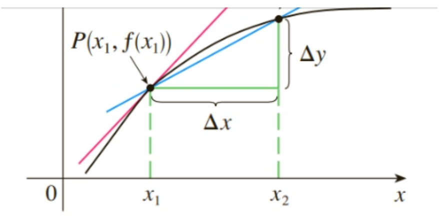
$= m_{tan} = f'(a)$

Most common way of thinking of it is as VELOCITY.

4 Definition The **derivative of a function f at a number a**, denoted by $f'(a)$, is

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

if this limit exists.



average rate of change $= m_{PQ}$

instantaneous rate of change $=$ slope of tangent at P

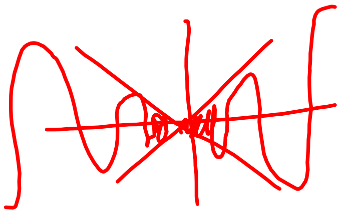
FIGURE 8

$$\Delta y = f(x_2) - f(x_1)$$

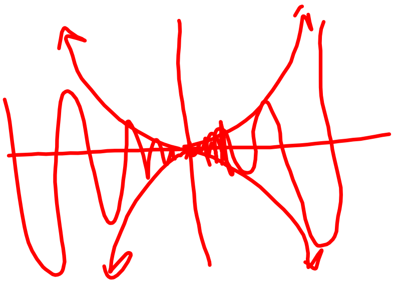
$$\frac{\Delta y}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

59–60 Determine whether $f'(0)$ exists.

59. $f(x) = \begin{cases} x\sin\dfrac{1}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$ No Yes



60. $f(x) = \begin{cases} x^2\sin\dfrac{1}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$



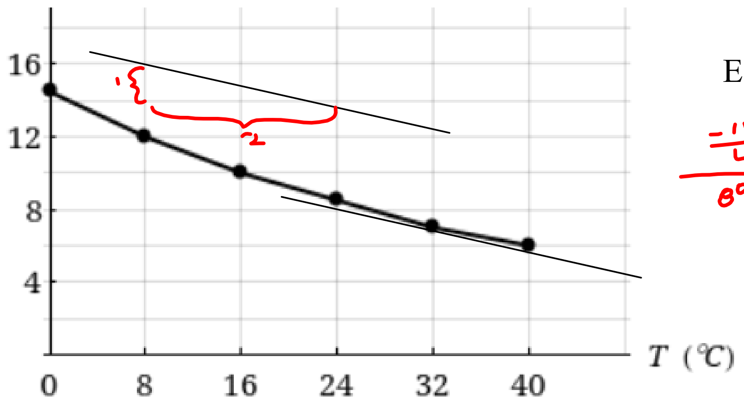
(a) $x = 0$

(59) $\dfrac{f(a+h)-f(a)}{h}$

$= \dfrac{f(0+h)-f(0)}{h} = \dfrac{h\sin(\frac{1}{h})-0}{h} = \dfrac{h\sin(\frac{1}{h})}{h} = \sin(\frac{1}{h})$ $(h \neq 0)$

$\xrightarrow{h\to 0}$ DNE. It oscillates between ± 1 infinitely often in any neighborhood of 0.

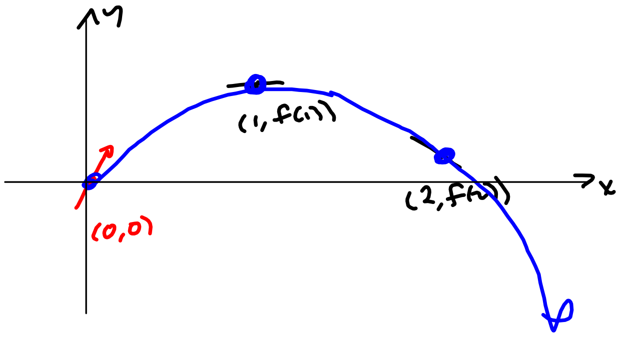
(60) ... $= h\sin(\frac{1}{h}) \xrightarrow{h\to 0} 0$ So $f'(0)$ exists!
$h \neq 0$



Estimate $S'(32)$

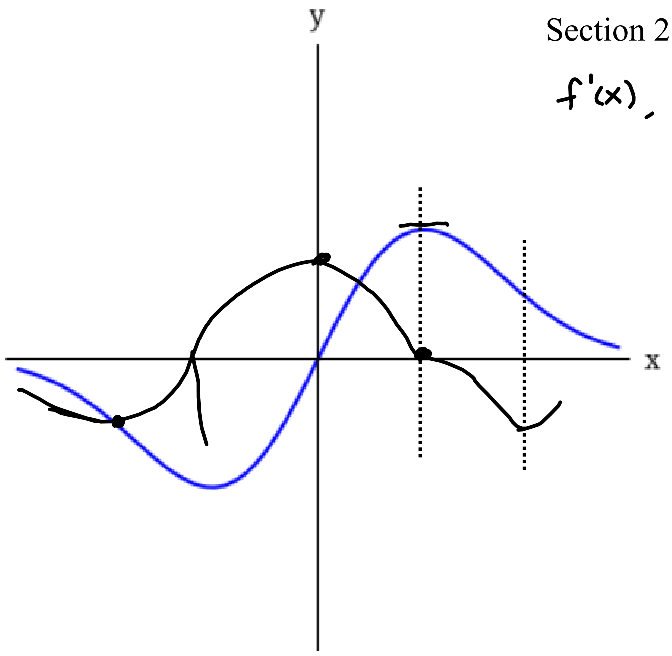
$\dfrac{-1\,\frac{mg}{L}}{8°C} \approx -\dfrac{1}{8}\,\dfrac{mg/L}{°C}$

Sketch the graph of a function f for which $f(0) = 0$, $f'(0) = 3$, $f'(1) = 0$, and $f'(2) = -1$.



Go directly to WebAssign.net. Don't depend on Aims server to access homework.

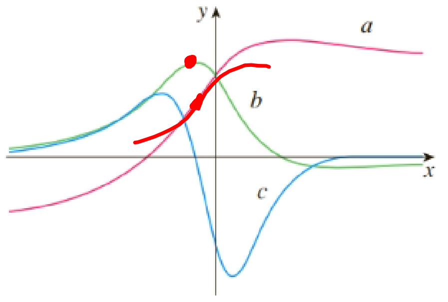
Use the given graph of $f(x)$ to sketch the graph of f'.



Section 2.2. Talking about

$f'(x)$, instead of just $f'(a)$

47. The figure shows the graphs of f, f', and f''. Identify each curve, and explain your choices.



$b = a'$?

$c = b'$

$a = c'$ → Nope

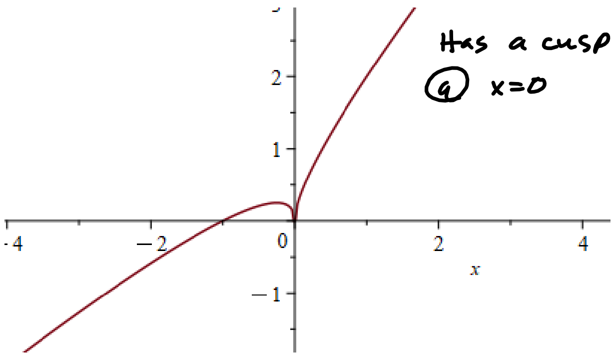
$c = a'$?

$c = f''$

$b = f'$

$a = f$

A graph of $f(x) = x + \sqrt{|x|}$



Has a cusp @ $x=0$

$$f(x) = \begin{cases} x + \sqrt{-x} & \text{if } x < 0 \\ x + \sqrt{x} & \text{if } x \geq 0 \end{cases}$$

$$f'(x) = \begin{cases} 1 - \frac{1}{2\sqrt{-x}} & \text{if } x < 0 \\ 1 + \frac{1}{2\sqrt{x}} & \text{if } x > 0 \end{cases}$$

$f'(0) \nexists$

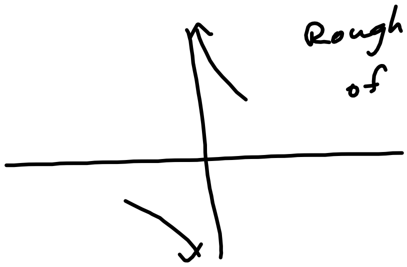
$y = x + \sqrt{-x} = x + (-x)^{\frac{1}{2}}$

$y' = 1 + \frac{1}{2}(-x)^{-\frac{1}{2}}(-1) = 1 - \frac{1}{2\sqrt{-x}}$ $\quad x < 0$

$y = x + \sqrt{x} = x + x^{\frac{1}{2}}$

$y' = 1x^0 + \frac{1}{2}x^{-\frac{1}{2}} = 1 + \frac{1}{2\sqrt{x}}$ $\quad x \geq 0$

$\lim\limits_{x \to 0} f'(x)$ DNE



Rough idea of y' near $x=0$