MAT 201-R41

Calculus I Schedule
Subject to Minor Revision

Spring, 20231
Page 1

Monday	Week	What we're doing this week:	Links
8/21/23	1	On Day 1, I will show you where things are on https://www.webassign.net/ and https://harryzaims.com/ . It's all set up so you can find MY take on almost every exercise I've assigned you, so I'm *with* you, 24/7, in a sense. Traditionally, I give you the freestyle pep talk on the differential calculus, proving to you that the slope of 2x is 2, which you know, and the slope of x^2 is 2x, which you likely don't. As time allows, I will work some of the dreary exercises in 1.1 and 1.2 that are meant to motivate the moves I make when I'm having fun with 2x and x^2. **** **** **** **** **** **Do Sections 1.1, 1.2*, 1.3** **Sections marked with * are 2-day sections. All others are 1-day sections. Each week we meet 4 times, but each week counts 5 lecture.**	WebAssign
8/28/23	2	**Last day to Drop is 8/25/23** **Sec 1.4. 1.5*, 1.6***	harryzaims.com
9/4/23	3	**1.7, 1.8** Review for Test 1	
9/11/23	4	**Test 1, Chapter 1, Open Dates: Monday, 9/11 thru Wednesday, 9/13** **2.1*, 2.2***	
9/18/23	5	**2.3*, 2.4** is given 3 days.	
9/25/23	6	2.5*, 2.6*, 2.7	

Planning a Writing Assignment over Chapter 1 next week.

Find the instantaneous slope of a function at a point by taking the limit of the difference quotient.

$$\lim_{x \to c} \frac{f(x)-f(c)}{x-c} = \lim_{h \to 0} \frac{f(c+h)-f(c)}{h}$$

c = 7 or something

$x = c+h$

$x-c = c+h-c = h$

Find the slope function for $f(x)$:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$$

x isn't fixed.

Find $f'(x)$ by the limit definition for $f(x) = \sqrt{x}$.

$$\frac{f(x+h)-f(x)}{h} = \frac{\sqrt{x+h}-\sqrt{x}}{h} = \left(\frac{\sqrt{x+h}-\sqrt{x}}{h}\right)\left(\frac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}}\right)$$

$$= \frac{x+h-x}{h(\sqrt{x+h}+\sqrt{x})} = \frac{h}{h(\sqrt{x+h}+\sqrt{x})} = \frac{1}{\sqrt{x+h}+\sqrt{x}} \xrightarrow{h \to 0} \frac{1}{\sqrt{x}+\sqrt{x}}$$

$(h \neq 0)$

$$= \boxed{\frac{1}{2\sqrt{x}} = f'(x)}$$

Baby-step build-up: Find the slope of $\sqrt{x}$ at $x = 3$

$$\frac{f(3+h)-f(3)}{h} = \left(\frac{\sqrt{3+h}-\sqrt{3}}{h}\right)\left(\frac{\sqrt{3+h}+\sqrt{3}}{\sqrt{3+h}+\sqrt{3}}\right)$$

$$\ldots = \frac{h}{h(\sqrt{3+h}+\sqrt{3})} = \frac{1}{\sqrt{3+h}+\sqrt{3}} \xrightarrow{h \to 0} \frac{1}{2\sqrt{3}}$$

$(h \neq 0)$

$a \sin(b(x-c)) + c$ is a transformed $\sin(x)$

Amplitude

vertical shift.

Phase shift $x-c$ RIGHT (Delay)

$x+c$ LEFT (Advance)

$bx = 2\pi$

when $x =$ period.

stretch/shrink

stretch/shrink i.e., $b = \frac{2\pi}{\text{period}}$

$a f(b(x-c)) + d \to y \mapsto y + d$ RIGID

($y-d$ is down d units)

$x \mapsto x + c$ RIGID

$y \mapsto 2y$ $x \mapsto \frac{1}{b}x$

Find an eq'n of the tangent line to

$f(x) = x^2 - 3x + 2$ @ the point $(3, 2)$

& sketch the picture of both

$$m_{sec} = \frac{f(3+h) - f(3)}{h} = \frac{(3+h)^2 - 3(3+h) + 2 - [3^2 - 3(3) + 2]}{h}$$

$$= \frac{9 + 6h + h^2 - 9 - 3h \boxed{+2 - 9 + 9 - 2}}{h} = \frac{3h + h^2}{h} = \frac{3+h}{1}$$

$h \neq 0$

$\xrightarrow{h \to 0} \boxed{3 = m_{tan}}$

$y = m_{tan}(x-3) + f(3)$

$\boxed{y = 3(x-3) + 2} = 3x - 9 + 2 = \boxed{3x - 7 = y}$

For Drill-and-Kill instructors.

$y = m(x - x_0) + y_0$ is point-slope for me.

$y - y_0 = m(x - x_0)$
Every Math Book.

In future

$f(x) = x^2 - 3x + 2$ find eq'n of tangent line @ $(3,2)$

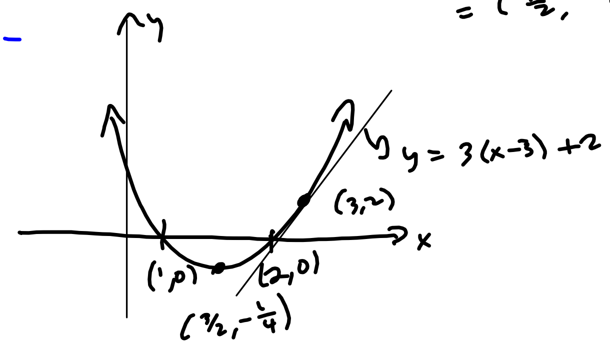
$\Rightarrow f'(x) = 2x - 3 \Rightarrow f'(3) = m_{tan} = 3 \Rightarrow$

$$\boxed{y = 3(x-3) + 2}$$

$(x-2)(x-1)$

vertex : $(1.5, -.25)$ $\quad \left(\frac{3}{2} - 2\right)\left(\frac{3}{2} - 1\right)$

$= \left(\frac{3}{2}, -\frac{1}{4}\right) \qquad = \left(-\frac{1}{2}\right)\left(\frac{1}{2}\right) = -\frac{1}{4}$



Prove the statement using the ε, δ definition of a limit.

$$\lim_{x \to 1} \frac{4+2x}{3} = 2$$

$y = \frac{4+2x}{3} = \frac{1}{3} + \frac{2}{3}x \Longrightarrow m = \frac{2}{3}$

$= \frac{2}{3}x + \frac{1}{3}$

$\Longrightarrow \delta = \frac{\varepsilon}{\frac{2}{3}} = \frac{3\varepsilon}{2}$ works!

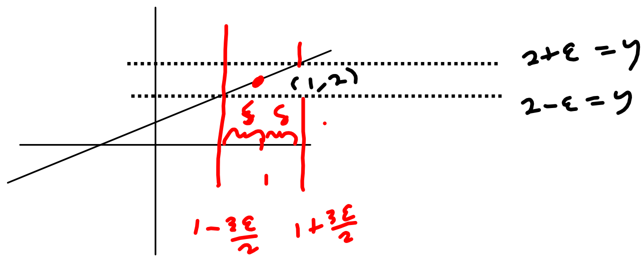
implies

Proof Let $\varepsilon > 0$. Define $\delta = \frac{\varepsilon}{\frac{2}{3}} = \frac{3\varepsilon}{2}$. Then $0 < |x-1| < \delta \Longrightarrow$

$\left| \frac{4+2x}{3} - 2 \right| = \left| \frac{4}{3} + \frac{2}{3}x - 2 \right| = \left| \frac{2}{3}x - \frac{2}{3} \right| = \frac{2}{3}|x-1| < \frac{2}{3}\delta$

$= \left(\frac{2}{3}\right)\left(\frac{3}{2}\varepsilon\right) = \varepsilon$ $\blacksquare$

$\frac{-6+4}{3} = -\frac{2}{3}$



Want $\frac{1}{(x+1)^4} > 10000 \implies$

$$\frac{1}{10000} > (x+1)^4$$

$$\sqrt[4]{\frac{1}{10^4}} > \sqrt[4]{(x+1)^4} = |x+1|$$

$$\left(\frac{1}{10^4}\right)^{\frac{1}{4}} = \frac{1^{\frac{1}{4}}}{(10^4)^{1/4}} = \boxed{\frac{1}{10} > |x+1|} \implies$$

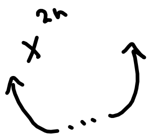
$$|x+1| < \frac{1}{10} \implies$$

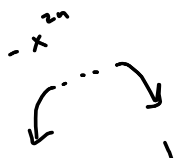
$$-\frac{1}{10} < x-(-1) < \frac{1}{10}$$

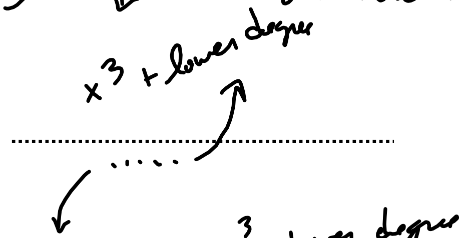
$x = x^3 + 1 \implies$

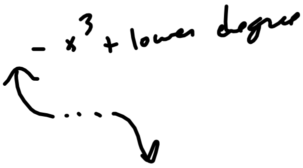
$x^3 - x + 1 = 0$ has a real solution

All odd degree polynomials have at least one real root.









IVT :

$(-5)^3 - (-5) + 1 = -125 + 6 < 0$

$2^3 - 2 + 1 = 8 - 1 > 0$

poly's cont.s

IVT applies.

$\exists\, c \in (-5, 2)$

$\ni f(c) = 0.$