Continuity for Practical Purposes:

Everything's continuous everywhere on its domain, so it comes down to domain, usually.

Intermediate Value Theorem:

If $f(x)$ is continuous on $[a,b]$, with $f(a)=L_a < f(b)=L_b$, and L is any real # such that $L_a < L < L_b \Rightarrow$ There exists a $c \in (a,b) \ni f(c)=L$.

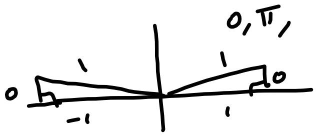
$\boxed{1.8\#17}$ Prove $\sin(x^3)=0$ has at least two distinct solutions in $(1,2)$

$$\sin(0^3)=0$$
$$\sin(1^3)=\sin(1) \approx 0.8414709848$$
$$\sin(2^3)=\sin(8) \approx 0.9893582466$$

$\sin(x^3)$ achieves all values of sine on $[0,2]$, since period of sine is $2\pi \approx 6.28$

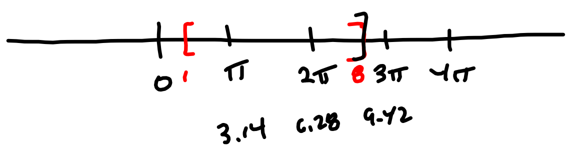
$$\sin(x)=0 \Rightarrow x \in \{n\pi \mid n \in \mathbb{Z}\}$$

$0, \pi,$



$0, \pm\pi, \pm 2\pi, \pm 3\pi, \ldots$

x^3 is running from 1 to 8.



They're putting the $\frac{3}{2}\pi$ in the list b/c $\sin(\frac{3\pi}{2})=-1$

Now, we use the Theorem

$$\left.\begin{array}{l} \sin(1^3) \approx .8415 > 0 \\ \sin(\frac{3\pi}{2}) = \sin\left(\left(\sqrt[3]{\tfrac{3\pi}{2}}\right)^3\right) = -1 \end{array}\right\} \Rightarrow \exists\, c \in \left(1, \sqrt[3]{\tfrac{3\pi}{2}}\right) \ni \sin(c)=0$$

$$\sin(2^3) = \sin(8) \approx .9894 > 0$$

$$\Rightarrow \exists\, c \in \left(\sqrt[3]{\tfrac{3\pi}{2}}, 2\right) \ni \sin(c^3)=0$$

#16 $\cos(x) - x^3 = 0$ is equivalent to

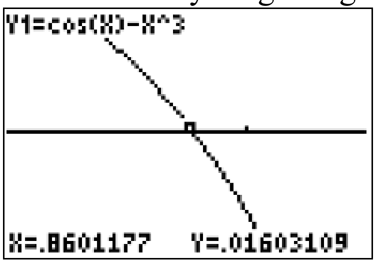
$\cos(x) = x^3$

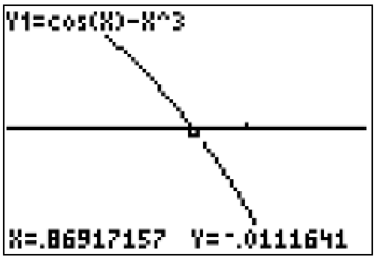
$f(x) = \cos(x) - x^3$

Note $\left.\begin{array}{l} \cos(0) - 0^3 = 1 - 0^3 = 1 > 0 \\ \cos(2) - 2^3 \leq 1 - 2^3 = -7 < 0 \end{array}\right\} \Rightarrow \exists\ c \in (0,2) \ni f(c) = 0$, by IVT.

Everything in sight is continuous on its domain.

Everything in sight has Domain = **R**.





$[.86, .87]$

Proof $\cos(x) = f(x)$ is cnts.

We show that $\lim_{x \to c} f(x) = f(c)$.

Let $x = a$.

Then as $h \to 0$, $a+h \to a$

so $\cos(a+h) = \cos(a)\cos(h) - \sin(a)\sin(h)$

$\xrightarrow{h \to 0} (\cos(a))(1) - (\sin(a))(0) = \cos(a)$

$$\lim_{h \to 0} (\cos(a+h)) = \lim_{h \to 0} [\cos(a)\cos(h) - \sin(a)\sin(h)]$$

$$= \ldots = \cos(a) \lim_{h \to 0} \cos(h) - \sin(a) \lim_{h \to 0} \sin(h)$$

$$= (\cos(a))(1) - (\sin(a))(0) = \cos(a)$$