Section 1.7 The Precise Definition of Limit

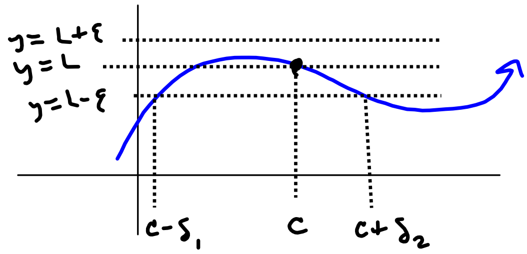
We find how close x needs to be to a limiting value to make y sufficiently close to a limiting y-value.

$|x-c| < \delta$ $\qquad$ $|f(x)-L| < \varepsilon$

Let $\varepsilon > 0$ be given. Find $\delta \ni$

If $|x-c| < \delta$, then $|f(x) - L| < \varepsilon$

It comes down to getting a bound or ceiling on how fast f(x) is changing with respect to x.



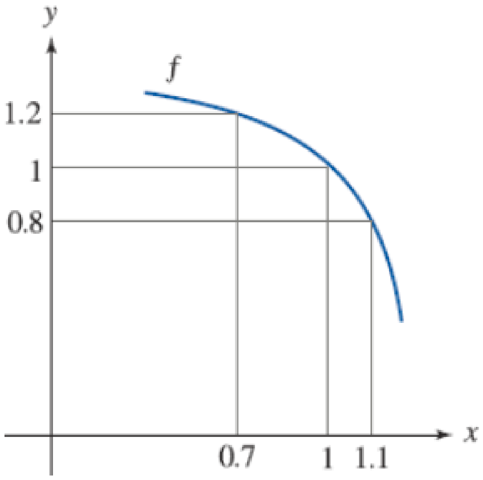
Here, we pick $\delta_2 \equiv \delta$, since it'll keep us inside the "ε-tube" if $c-\delta < x < c+\delta$,

i.e. $-\delta < x-c < \delta$

i.e. $|x-c| < \delta$

Use the given graph of f to find a number δ such that if $|x - 1| < \delta$ then $|f(x) - 1| < 0.2$.

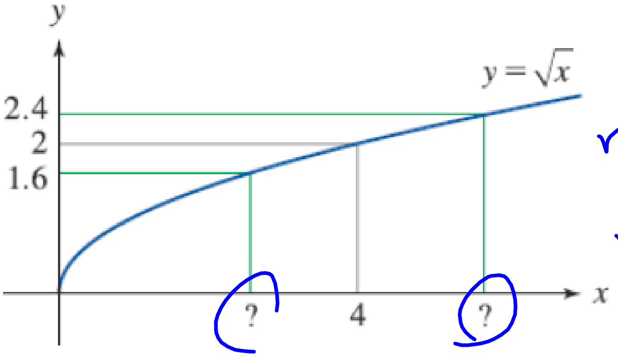
1.7 #1



$\boxed{\delta = 0.1}$

Use the given graph of $f(x) = \sqrt{x}$ to find a number δ such that if $|x - 4| < \delta$ then $\left|\sqrt{x} - 2\right| < 0.4$.

1.7 #3



$\sqrt{x} = 2.4$

$x = 5.76$

$\sqrt{x} = 1.6$

$x = 2.56$

$5.76 - 4 = 1.76$

$4 - 2.56 = 1.44$

smaller!

Do this algebraically:

Want $|\sqrt{x} - 2| < .4$

$-.4 < \sqrt{x} - 2 < .4$

$1.6 < \sqrt{x} < 2.4$

$(1.6)^2 < \sqrt{x}^2 < 2.4^2$

$2.56 < x < 5.76$

$4 - 2.56 = 1.44$ smaller.

$5.76 - 4 = 1.76$

1.7 #6

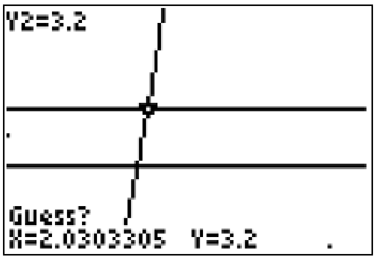
A graphing calculator is recommended.

For the limit $\lim_{x \to 2} (x^3 - 5x + 5) = 3$, illustrate this definition by finding the largest possible values of δ that correspo

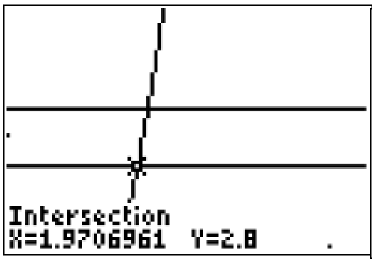
$\varepsilon = 0.2$ and $\varepsilon = 0.1$. (Round your answers to four decimal places.)

$\varepsilon = 0.2$ δ =

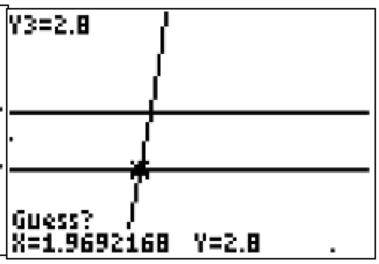
$\varepsilon = 0.1$ δ =



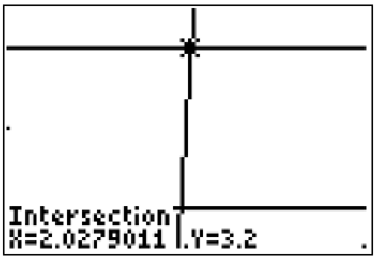
$\delta_2 = .0303305$



.029?

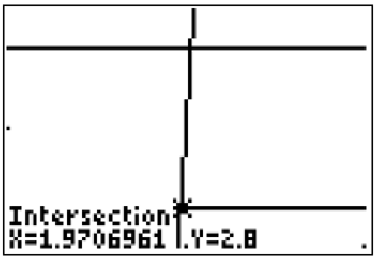


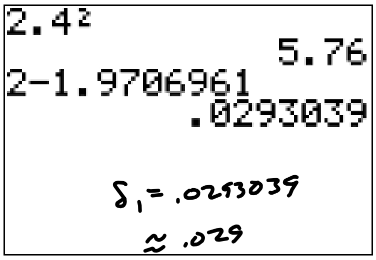
Second Attempt (Getting smaller) Ans. was 0.028



$\delta_2 = .0279011$

$\approx .028$





$\delta_1 = .0293039$

$\approx .029$

A machinist is required to manufacture a circular metal disk with area 1200 cm^2.

(a) What radius produces such a disk? (Round your answer to four decimal places.)

(b) If the machinist is allowed an error tolerance of ±7 cm^2 in the area of the disk, how close to the ideal radius in part (a) must the machinist control the radius? (Round your answers to four decimal places.)

____ cm < r < ____ cm

(c) In terms of the ε, δ definition of $\lim_{x \to a} f(x) = L$, what is x?

- ○ area
- ○ target radius
- ○ radius
- ○ target area

1.7 #7

```
√(1200/π)
        19.54410048
```

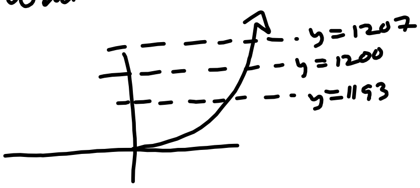
(a) $\pi r^2 = 1200 \text{ cm}^2$

$r^2 = \frac{1200}{\pi}$

$r = \sqrt{\frac{1200}{\pi}} \approx 19.5442 \text{ cm}$

$\varepsilon = 7 \text{ cm}^2$

Want $|\pi r^2 - 1200| < 7$



Method:
Solve $\pi r^2 = 1207$ (A)
& $\pi r^2 = 1193$ (B)

(A) $r^2 = \frac{1207}{\pi}$

$r = \sqrt{\frac{1207}{\pi}}$

(B) $r = \sqrt{\frac{1193}{\pi}}$

```
√(1200/π)
        19.54410048
√(1207/π)
        19.60102121
√(1193/π)
        19.48701348
```

$$\begin{array}{r} 19.60102121 \\ -\ 19.54410048 \\ \hline .05692073 \end{array}$$

$$\begin{array}{r} 19.54410048 \\ -\ 19.48701348 \\ \hline \end{array}$$

```
        19.48701348
√(1193/π)-√(1200
/π)
          -.057087
√(1207/π)-√(1200
/π)
      .0569207377
```

$\delta_1 \approx .057087 \approx .0571$

$\delta_2 \approx .0569207377$

$\approx .0569$

Linear Proofs:

Let $\delta = \frac{\varepsilon}{m}$

Claim: $\lim_{x \to 7} (2x-5) = 9$

Proof Let $\varepsilon > 0$ be given. Then if $\delta \equiv \frac{\varepsilon}{2}$, then for all x such that $0 < |x-7| < \delta$, we have

$$|2x-5-9| = |2x-14| = 2|x-7| < 2\delta = 2\left(\frac{\varepsilon}{2}\right) = \varepsilon \blacksquare$$

$\forall$ = "for all, for each, for every."

$\ni$ = "such that, so that"

$\exists$ = "There is."

Let $\varepsilon > 0$. Then $\forall\, x \ni |x-7| < \delta \equiv \frac{\varepsilon}{2}$, we have

$$|2x-5-9| = |2x-14| = 2|x-7| < 2\delta = 2 \cdot \frac{\varepsilon}{2} = \varepsilon \blacksquare$$