Domain of $f \circ g$?

NOT YET

$$\frac{f(x+h)-f(x)}{h} \xrightarrow{h \to 0} f'(x)$$

$\lim_{x \to c} f(x) = L$ means I can make $f(x)$ as close to L as I want by taking x sufficiently close to c.

$\lim_{x \to c} f(x) = L$ means Given $\varepsilon > 0$, there is a $\delta > 0$ such that if $0 < |x-c| < \delta$ then $|f(x) - L| < \varepsilon$

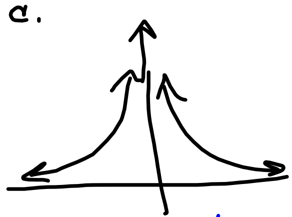
(FIND δ)

f is continuous at $x=c$ if $\lim_{x \to c} f(x) = f(c)$

Infinite Limit (In some settings we say $\lim f$ DNE) $\nexists$

$\lim_{x \to c} f(x) = \infty$ means, given $N > 0$,
we can make all values of $f(x) > N$, by taking
x sufficiently close to c.

$$\lim_{x \to 0} \frac{1}{x^2} = \infty$$



Let $N = 10{,}000$
I can make $\frac{1}{x^2} > N$ for all x "close" to $x = 0$.

Scratch:

Want $\frac{1}{x^2} > 10{,}000$

$1 > 10{,}000\, x^2$

$10{,}000\, x^2 < 1$

$x^2 < \frac{1}{10000}$

$|x| = \sqrt{x^2} < \sqrt{\frac{1}{10000}} = \sqrt{\frac{1}{10^4}} = \frac{\sqrt{1}}{\sqrt{10^4}} = \frac{1}{100}$

$|x| < \frac{1}{100} \Rightarrow$

$-\frac{1}{100} < x < \frac{1}{100}$

& for all x in this "band" about $x = 0$,
except @ $x = 0$ $\frac{1}{x^2} > 10{,}000$.

Prove:

Let $N > 0$ be given.

Scratch Want $\frac{1}{x^2} > N$

$1 > Nx^2$

$x^2 < \frac{1}{N}$

$|x| = |x - 0|$ $|x| < \sqrt{\frac{1}{N}} = \frac{1}{\sqrt{N}} = \delta$!

$-\frac{1}{\sqrt{N}} < x < \frac{1}{\sqrt{N}}$ This gives us δ!

Proof Let $N > 0$ be given. Then if $\delta = \frac{1}{\sqrt{N}}$, we have

$0 < |x - c| < \delta$ implies

$$\frac{1}{x^2} = \frac{1}{|x-0|^2} = \left(\frac{1}{|x-0|}\right)^2$$

$$> \left(\frac{1}{\frac{1}{\sqrt{N}}}\right)^2 = \left(\sqrt{N}\right)^2 = N$$

$A > 0$ & $A < 5$ Then

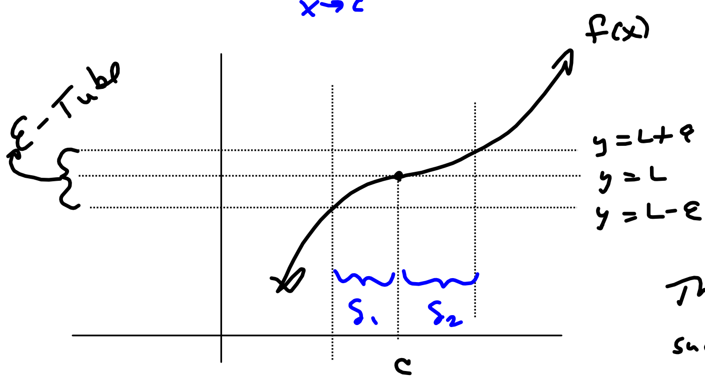
$\frac{1}{5} < \frac{1}{A}$

$\frac{1}{A} > \frac{1}{5}$

$5 < 6$

$\frac{1}{6} < \frac{1}{5}$

Picture for $\lim_{x \to c} f(x) = L$



The idea is to find $\delta > 0$ such that

$|f(x) - L| < \varepsilon$, i.e.

$-\varepsilon < f(x) - L < \varepsilon \rightsquigarrow L - \varepsilon < f(x) < L + \varepsilon$

$|x - c| < \delta = \delta_1$ for this picture.

Topologist's SINE CURVE

$f(x) = \sin\left(\frac{\pi}{x}\right)$ oscillates an ∞ # of times in any neighborhood of $x=0$.

It's bounded, but it's pathological @ $x=0$.

Limit $\nexists$ @ $x=0$.

FACTORING REVIEW

$x^3-y^3 = (x-y)(x^2+xy+y^2)$

$x^3+y^3 = (x+y)(x^2-xy+y^2)$

$x^2-y^2 = (x+y)(x-y)$

Consider $f(x) = \frac{x^3-8}{x^2-4}$

Find $\lim\limits_{x\to 2} f(x)$ I can't plug in $x=2$, at least not right away.

$$\frac{x^3-8}{x^2-4} = \frac{\cancel{(x-2)}(x^2+2x+2^2)}{(x+2)\cancel{(x-2)}} = \frac{x^2+2x+4}{x+2} \xrightarrow{x\to 2} \frac{2^2+2\cdot 2+4}{2+2}$$

$(x \neq 2)$

$$= \frac{12}{4} = 3$$

Book Way:

$$\lim_{x\to 2} \frac{x^3-8}{x^2-4} = \lim_{x\to 2} \frac{(x-2)(x^2+2x+4)}{(x-2)(x+2)} = \lim_{x\to 2} \frac{x^2+2x+4}{x+2}$$

$$= \cdots = 3.$$

Training on Attacking the material the first time?

The homework videos are called "Heuristic Learning."

You know what you're trying to do before you do the learning.

Not like "Wax On, Wax Off" in "Karate Kid."