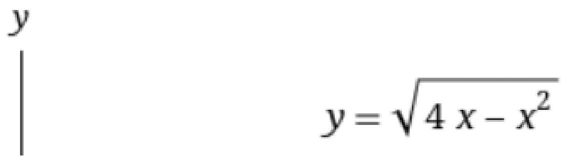
Section 1.3 #3

The graph of $y = \sqrt{4x - x^2}$ is shown in the figure.



$$4x - x^2 = -(x^2 - 4x \quad) = -(x^2 - 4x + 2^2 - 4)$$

$$\Rightarrow \quad y = \sqrt{-(x-2)^2 + 4}$$

$$= \sqrt{4 - (x-2)^2} \quad \text{Hopefully, you see this as}$$

$y = \sqrt{4 - x^2}$ shifted right 2 units

Hopefully, you see this as the top half of a circle of radius 2, centered at the origin.

$x^2 + y^2 = r^2$ $\quad x^2 + y^2 = 4$ is eq'n of circle of radius $r = 2$, centered @ the origin.

$$y^2 = 4 - x^2$$

$$|y| = \sqrt{y^2} = \sqrt{4 - x^2}$$

$$|y| = \sqrt{4 - x^2}$$

$$y = \pm\sqrt{4 - x^2}$$

is square-root property

$A^2 = B \Rightarrow$

$A = \pm\sqrt{B}$

Top half

$y = \sqrt{4 - x^2}$

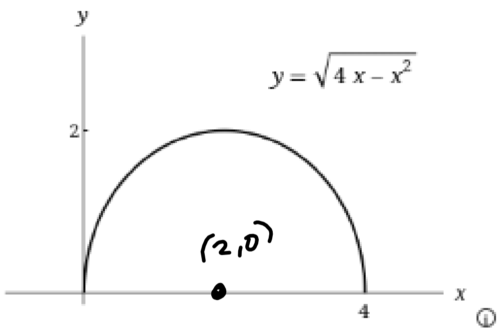
$\Downarrow$

$y = \sqrt{4 - (x-2)^2}$ is shifted right 2.

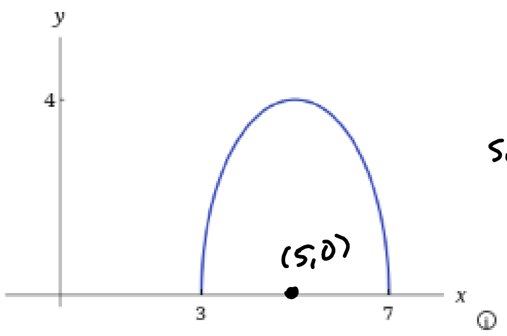
Bottom half

$y = -\sqrt{4 - x^2}$

The graph of $y = \sqrt{4x - x^2}$ is shown in the figure.



Use transformations to construct a function whose graph is as shown.



So it looks like:
vertical stretch
by factor of 2
& horizontal shift
by 3 units.

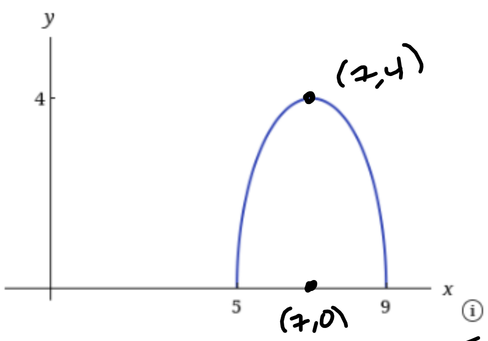
$y = \sqrt{4x - x^2}$ &

Stretch $y = 2\sqrt{4x - x^2}$

Shift $y = 2\sqrt{4(x-3) - (x-3)^2}$

How about this one?

Use transformations to construct a function whose graph is as shown.



RIGHT 5

vertical Stretch $y \mapsto 2y$

$$y = 2\sqrt{4(x-5) - (x-5)^2}$$
$$= 2\sqrt{4x-20-(x^2-10x+25)}$$
$$= 2\sqrt{4x-2-x^2+10x-25}$$
$$= 2\sqrt{-x^2+14x-27}$$

A variable star is one whose brightness alternately increases and decreases. For one such star, the time between periods of maximum brightness is 4.6 days, the average brightness (or magnitude) of the star is 5.4, and its brightness varies by ±0.25 magnitude. Find a function that models the brightness of the star as a function of time (in days), t. (Assume that at $t = 0$ the brightness of the star is 5.4 and that the function is increasing.)

T = period = 4.6 days

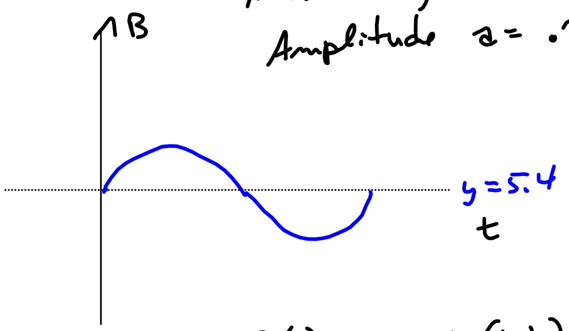
midline $y = 5.4$

Amplitude $a = .25$

t = # of days

Assume

B = Brightness $= 5.4$ (midline) @ $t = 0$, i.e., $B(0) = 5.4$



$B(t) = .25 \sin(bt) + 5.4$

$bt = 2\pi$ when $t = 4.6$

$b = \frac{2\pi}{4.6}$

$B(t) = .25 \sin\left(\frac{2\pi}{4.6}t\right) + 5.4$

Find each of the following functions and state their domains. (Enter the domains in interval notation.)

$f(x) = \sqrt{5 - x}, \quad g(x) = \sqrt{x^2 - 4}$

(a) $f + g$

$f + g =$ (No Response)
domain (No Response)

(b) $f - g$

$f - g =$ (No Response)
domain (No Response)

(c) fg

$fg =$ (No Response)
domain (No Response)

(d) $\frac{f}{g}$

$\frac{f}{g} =$ (No Response)
domain (No Response)

$\mathcal{D}(f)$:

Need $5 - x \geq 0$

$5 \geq x$

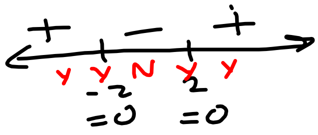
$\mathcal{D}(f) = \{ x \mid x \leq 5 \}$

$= (-\infty, 5]$

$\mathcal{D}(g)$:

Need $x^2 - 4 \geq 0$

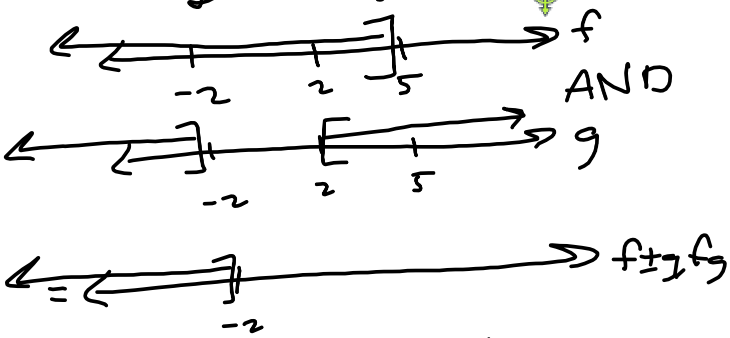
$(x-2)(x+2) \geq 0$



$\mathcal{D}(g) = (-\infty, -2] \cup [2, \infty)$

$\mathcal{D}(f \pm g) = \mathcal{D}(fg)$

$= \mathcal{D}(f) \cap \mathcal{D}(g)$



For $\frac{f}{g}$, we need $g(x) \neq 0$.

(Throw out $x = \pm 2$

b/c $g(\pm 2) = 0$

$\mathcal{D}(f \pm g) = \mathcal{D}(fg) = \mathcal{D}(f) \cap \mathcal{D}(g) = \{ x \mid x \in \mathcal{D}(f) \cap \mathcal{D}(g) \}$

$= \{ x \mid x \in \mathcal{D}(f) \text{ and } x \in \mathcal{D}(g) \}$

$\mathcal{D}\left(\frac{f}{g}\right) = \{ x \mid x \in \mathcal{D}(f) \cap \mathcal{D}(g) \text{ AND } g(x) \neq 0 \}$

$\sqrt{\text{Negative}}$ Bad

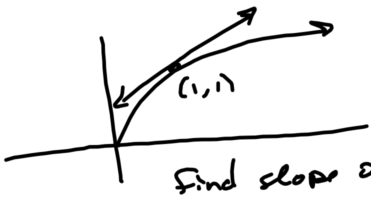
$\frac{\text{Stuff}}{0}$ BAD

§ 1.6 #12

Evaluate the limit, if it exists. (If an answer does not exist, enter DNE.).

$$\lim_{h \to 0} \frac{\sqrt{1+h} - 1}{h}$$

1/2



$f(x) = y = \sqrt{x}$

Find slope of $y = \sqrt{x}$ @ $x = 1$

$$\lim_{x \to 1} \frac{f(x) - f(1)}{x - 1}$$

$$= \lim_{h \to 0} \frac{f(1+h) - f(1)}{h}$$

$$= \lim_{h \to 0} \frac{\sqrt{1+h} - \sqrt{1}}{h}$$

Lauren's version:

$$\lim_{h \to 0} \frac{\sqrt{9+h} - 3}{h}$$

$$\frac{\sqrt{9+h} - 3}{h} = \left(\frac{\sqrt{9+h} - 3}{h}\right)\left(\frac{\sqrt{9+h} + 3}{\sqrt{9+h} + 3}\right) = \frac{9+h-9}{h\left(\sqrt{9+h} + 3\right)}$$

$$= \frac{h}{h\left(\sqrt{9+h} + 3\right)} = \frac{1}{\sqrt{9+h} + 3} \xrightarrow{h \to 0} \frac{1}{\sqrt{9} + 3} = \frac{1}{3+3}$$

$(h \neq 0)$

$= \frac{1}{6}$ = slope of $\sqrt{x}$ @ $x = 9$

Section 1.6 #2: Assume $\lim f$ & $\lim g$ exist.

then $\lim (f \pm g) = \lim f \pm \lim g$

$\lim (fg) = (\lim f)(\lim g)$

$\lim \left(\frac{f}{g}\right) = \frac{\lim f}{\lim g}$, provided $\lim g \neq 0$