Section 1.3 #11 We build a cosine function from data.

Hours in varied latitudes

	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
20°	12	12.3	12.9	13	12.8	12.5	12	11.6	11	10.9
30°	12	13.2	13.7	14	13.8	12.8	12	11.2	10.2	10
40°	12	13.5	14.3	14.9	14.2	13.2	12	10.8	9.7	9.1
50°	12	14	15.5	16.1	15.7	14	12	10	8.3	7.9
60°	12	14.9	17.7	18.2	17.8	15	12	9	6.5	5.7

$\boxed{2\sin\left(\frac{2}{365}\pi(t-80)\right)+12}$

New Orleans is 30 degrees North. We model its length of day as a function

Jan 31
Feb 28
Mar 31

90 days

Equinoxes:
Mar 21
June 21

Mar 21^{st} is 80^{th} day

start (a) $x = 80$

Period: 365 days

sine has period 2π

$\sin(bx)$

$bx = 2\pi$ when $x = 365$

$\Rightarrow 365b = 2\pi$

$b = \frac{2\pi}{365}$

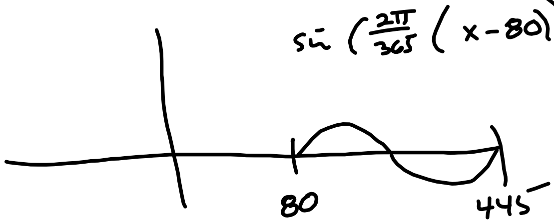
$\sin\left(\frac{2\pi}{365}(x)\right)$

$\sin(x)$

$\sin\left(\frac{2\pi}{365}x\right)$

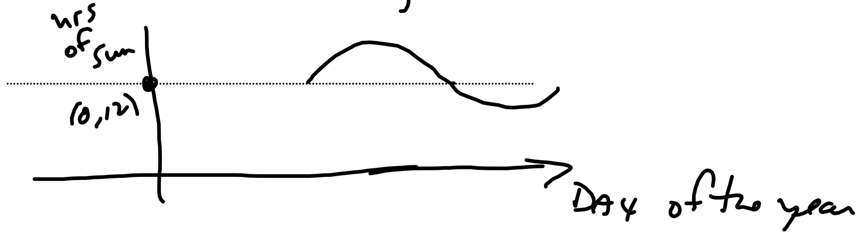
Start @ $x = 80$

$\sin\left(\frac{2\pi}{365}(x-80)\right)$



Midline is $y = 12$ hours

$y = \sin\left(\frac{2\pi}{365}(x-80)\right) + 12$

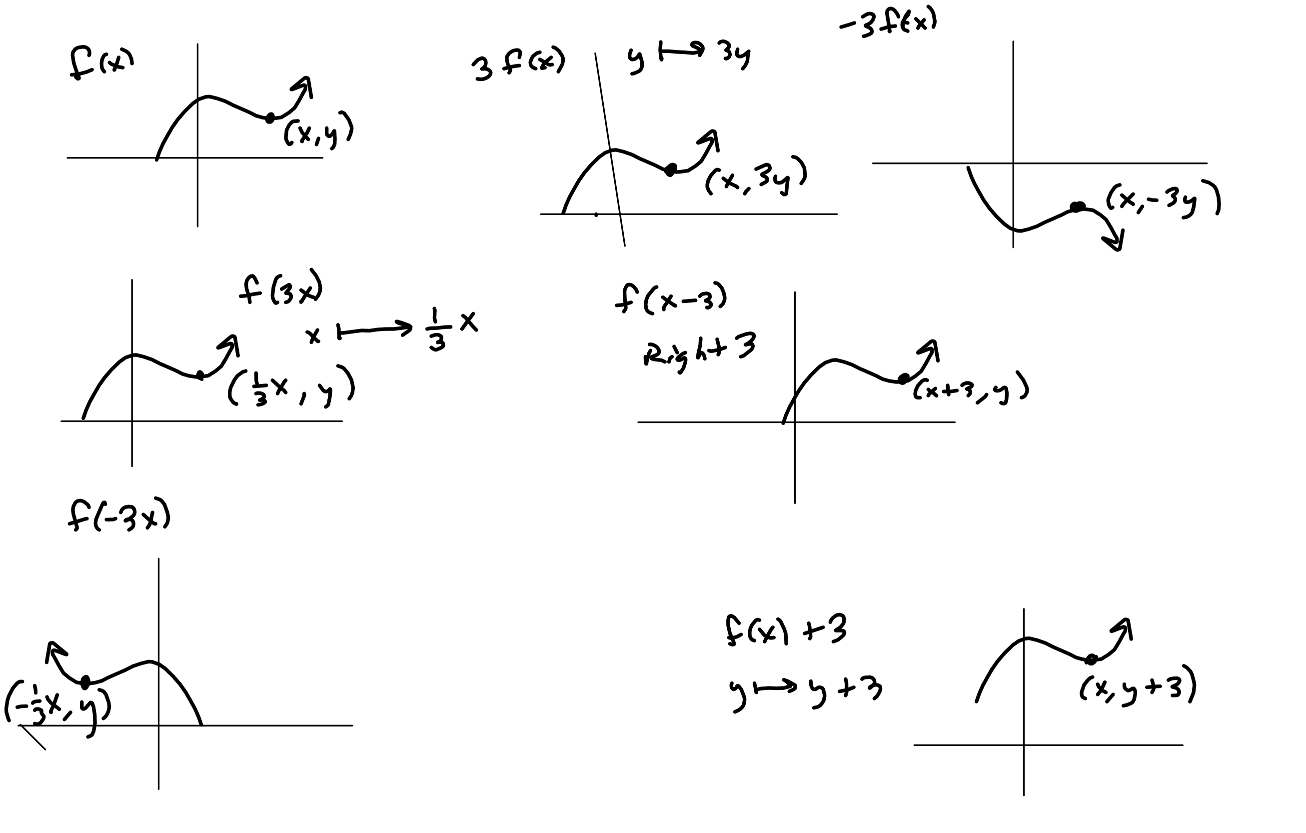


Now for the amplitude: Find maximum departure from y = 12

Hours of sun vary from 10 up to 14 hours. That's a departure of 2 hours above and below the mid-line y = 12

$y = 2\sin\left(\frac{2\pi}{365}(x-80)\right) + 12$

$y = f(x)$, (x, y) on graph of $f(x)$



Building a sine or cosine from data:

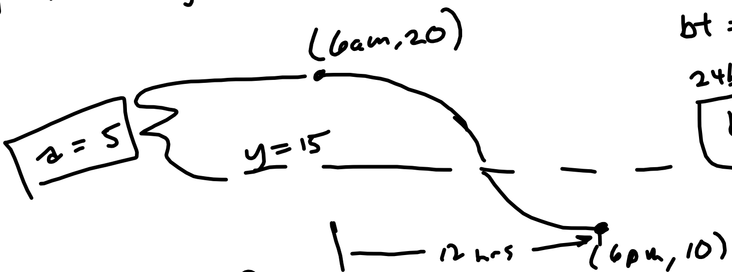
Cosine preferred, because all you need is the period, amplitude and high and low.

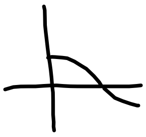
Tides: High is 20 ft @ 6am
Low is 10 ft @ 6pm

$a\cos(b(t-c))+d$

t = time in hours after midnight
y = tide height in feet

Period = 24hr
$bt = 2\pi$ when $t = 24$
$24b = 2\pi$
$\boxed{b = \frac{2\pi}{24} = \frac{\pi}{12}}$





High @ 6am = $t=6$
cosine starts @ its high point. we delay by 6 hours
$\boxed{c=6}$ $t-6$ inside

Midline: $y = \boxed{15 = d}$

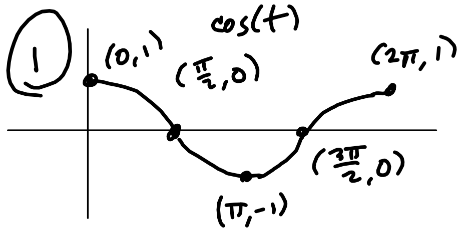
$$\boxed{f(t) = 5\cos\left(\frac{\pi}{12}(t-6)\right) + 15}$$

$f(t) = 5\cos\left(\frac{\pi}{12}(t-6)\right) + 15$

We graph this by shifting and stretching cosine.

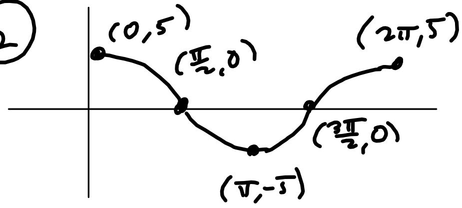
1

$\cos(t)$



$5\cos(t)$

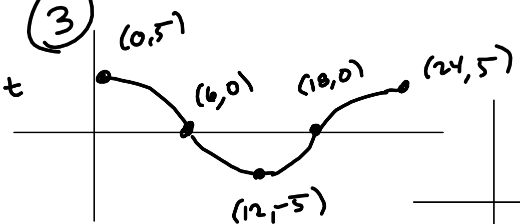
2



$5\cos\left(\frac{\pi}{12}t\right)$

3

$t \longmapsto \frac{12}{\pi}t$



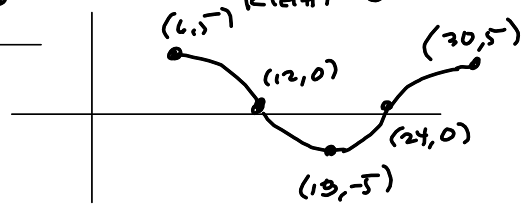
$\frac{12}{\pi}\cdot\frac{\pi}{2} = 6$

$\frac{12}{\pi}\cdot\pi = 12$

4

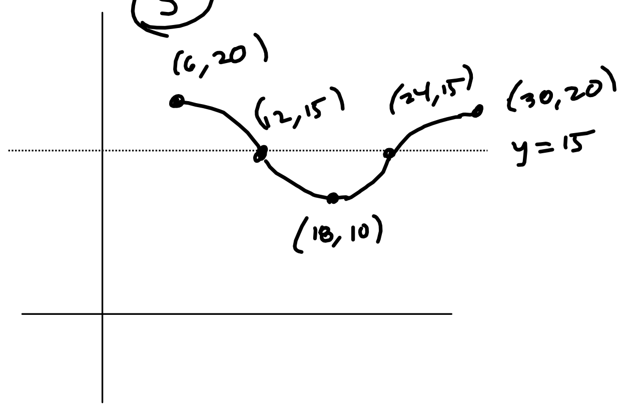
$5\cos\left(\frac{\pi}{12}(t-6)\right)$

RIGHT 6



Midline is $y=15$

5



You can swap steps 3 & 4. Sometimes that helps avoid messy fractions, but I prefer to see you do your stretching before your work-out.

A solved version of 1.3 #12. Try it on and ask if you still have questions.

12. 0/1 points SCalc9 1.3.028. [4708222]

A variable star is one whose brightness alternately increases and decreases. For one such star, the time between periods of maximum brightness is 3.8 days, the average brightness (or magnitude) of the star is 4.3, and its brightness varies by ±0.40 magnitude. Find a function that models the brightness of the star as a function of time (in days), t. (Assume that at $t = 0$ the brightness of the star is 4.3 and that the function is increasing.)

$f(t) =$ $\boxed{4.3 + 0.4\sin\left(\frac{(2\pi)t}{3.8}\right)}$

Solution or Explanation

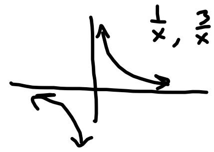
Using a sine function to model the brightness of the star as a function of time, we take its period to be 3.8 days, its amplitude to be 0.4 (on the scale of magnitude), and its average magnitude to be 4.3. If we take $t = 0$ at a time of average brightness, then the magnitude (brightness) as a function of time t in days can be modeled by the formula

$$f(t) = 4.3 + 0.4\sin\left(\frac{2\pi}{3.8}t\right).$$

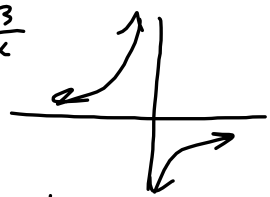
$$f(x)=\frac{3}{6-x}\Rightarrow m_{PQ}=\frac{f(x)-f(7)}{x-7}=\frac{\frac{3}{6-x}-(-3)}{x-7}$$

$$\frac{3}{6-x}=\frac{-3}{x-6}$$

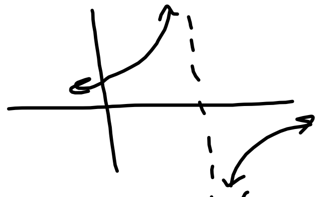
$\frac{1}{x}$, $\frac{3}{x}$



$-\frac{3}{x}$

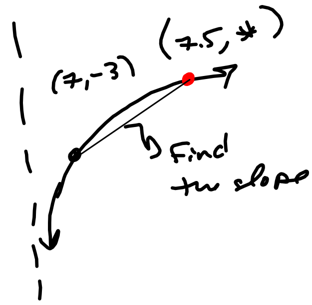


$$\frac{-3}{x-6}$$



$x=6$

In the vicinity of $x=7$



Slope of the line connecting $(x,f(x))$ & $(7,f(7))$ is $\frac{f(x)-f(7)}{x-7}=\frac{\frac{3}{6-x}-\frac{3}{6-7}}{x-7}$

is the difference quotient

S 1.4 is ONLY TO MOTIVATE the use of limits! That limits kind of DO work.

In the sequel, we find the slope of $y=\frac{3}{6-x}$ @ $x=7$ as follows: $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ & then plug in $x=7$

$$f'(7)=\lim_{x\to 7}\frac{f(x)-f(7)}{x-7}$$

$$\frac{\frac{3}{6-(x+h)}-\frac{3}{6-x}}{h}=\frac{1}{h}\left[\frac{3}{6-(x+h)}\cdot\left(\frac{6-x}{6-x}\right)-\left(\frac{3}{6-x}\right)\left(\frac{6-(x+h)}{6-(x+h)}\right)\right]$$

$$\frac{1}{h}\left(\frac{18-3x-18+3x+3h}{(6-(x+h))(6-x)}\right)=\frac{1}{h}\left(\frac{3h}{(6-(x+h))(6-x)}\right)$$

$$=\frac{3}{(6-(x+h))(6-x)}\xrightarrow{h\to 0}\frac{3}{(6-x)(6-x)}$$

$$=\frac{3}{(6-x)^2}=f'(x)$$

What's $f'(7)$?

$$\frac{3}{(6-7)^2}=\frac{3}{(-1)^2}=\boxed{3=m}$$