51–56 Find an expression for the function whose graph is the given curve.

51. The line segment joining the points $(3,-5)$ & $(7,1)$
(x_1,y_1) (x_2,y_2)

$$m = \frac{y_2-y_1}{x_2-x_1} = \frac{1-(-5)}{7-3} = \frac{6}{4} = \frac{3}{2}$$

$$y = \frac{3}{2}(x-3)-5$$
$$= \frac{3}{2}x - \left(\frac{3}{2}\right)\left(\frac{3}{1}\right) - 5$$
$$= \frac{3}{2}x - \frac{9}{2} - \frac{5}{1}\cdot\frac{2}{2} = \frac{3}{2}x - \frac{9}{2} - \frac{10}{2}$$
$$= \frac{3}{2}x - \frac{19}{2}$$

WebAssign

$$\frac{3x}{2} - \frac{19}{2}$$

33. 0/2 points SCalc9 1.1.066. [4708220]

Find a formula for the described function.

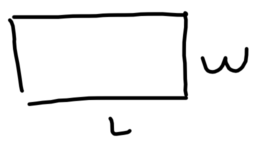
A rectangle has area 16 m^2. Express the perimeter P (in m) of the rectangle as a function of the length L of one of its sides.

$P(L) =$ [ ] ✖ $2L + \frac{32}{L}$ m

State the domain of P. (Assume the length of the rectangle is longer than its width. Enter your answer using interval notation.)

[ ] ✖ $(4, \infty)$

$\boxed{LW = 16}$ Area $\rightarrow$ $W = \frac{16}{L}$



Perimeter is $2L + 2W$

$P = 2L + 2W = 2L + 2\left(\frac{16}{L}\right) = 2L + \frac{32}{L}$

Need $LW = 16$

$(100{,}000{,}000)\left(\frac{16}{100{,}000{,}000}\right) = 16$

We can make the length L as big as we want, but if it's to be the longer of the sides, then it can't be smaller than 4.

In Lauren's case, Area was 49, so L > 7 and can be made as large as we wish, and the area will be LW = L(49/L) = 49

"OIC"

16. 0/1 points SCalc9 1.1.522.XP. [4758501]

A spherical balloon with radius r inches has volume $V(r) = \frac{4}{3}\pi r^3$. Find an expression that represents the amount of air required to inflate the balloon from a radius of r inches to a radius of $r + 3$ inches. (Express your answer in terms of π and r.)

$\boxed{\frac{4}{3}\pi\left(9r^2 + 27r + 27\right)}$

$V = \frac{4}{3}\pi r^3$

Change in volume between a sphere of radius r_1 to a radius of r_2 would be

$$\frac{4}{3}\pi r_2^3 - \frac{4}{3}\pi r_1^3$$

$$= \frac{4}{3}\pi (r_1+3)^3 - \frac{4}{3}\pi r_1^3$$

$$(A+B)^3 = A^3 + 3A^2B + 3AB^2 + B^3$$

$$r^3 + 3r^2(3) + 3r(3)^2 + 3^3$$

$$= r^3 + 9r^2 + 27r + 27$$

$\Rightarrow$ Volume =

$$= \frac{4}{3}\pi(r+3)^3 - \frac{4}{3}\pi r^3 = \frac{4}{3}\pi\left[(r+3)^3 - r^3\right]$$

$$= \frac{4}{3}\pi\left[r^3 + 9r^2 + 27r + 27 - r^3\right]$$

$$= \frac{4}{3}\pi\left[9r^2 + 27r + 27\right]$$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1

Ask about Binomial Theorem and Pascal's Triangle

17. 0/1 points

Evaluate the difference quotient for the given function. Simplify your answer.

$$f(x) = 3 + 5x - x^2, \quad \frac{f(3+h) - f(3)}{h}$$

$-h - 1$

$(a+b)^2 = (a+b)(a+b)$

$= a^2 + ab + ba + b^2 = a^2 + ab + ab + b^2$

$= a^2 + 2ab + b^2$

$f(3) = 3 + 5(3) - (3)^2 = 18 - 9 = \boxed{9 = f(3)}$

$f(3+h) = 3 + 5(3+h) - (3+h)^2$

$= 3 + 15 + 5h - (3^2 + 2(3)h + h^2)$

$= 18 + 5h - 9 - 6h - h^2$

$= \boxed{9 - h - h^2 = f(3+h)}$

$\Rightarrow \frac{f(3+h) - f(3)}{h} = \frac{9 - h - h^2 - 9}{h} = \frac{-h - h^2}{h} =$

$\frac{h(-1-h)}{h} = -1 - h$

Recall Completing the square:

$$x^2+bx = x^2+bx+\left(\frac{b}{2}\right)^2-\frac{b^2}{4} = \left(x+\frac{b}{2}\right)^2-\frac{b^2}{4}$$

$$4x-x^2 = -\left(x^2-4x \quad\right)$$

$$= -\left(x^2-4x+2^2-2^2\right)$$

$$\frac{4}{2}=2 \rightsquigarrow 2^2$$

$$= -\left(x^2-4x+2^2\right)+4$$

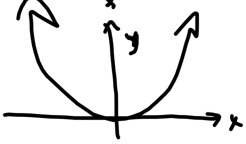
Graph it $\quad = -(x-2)^2+4$

using $f(x)=x^2$ as a model & transform it

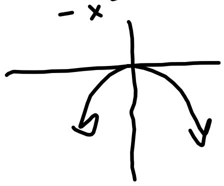
Delay!

$f(x-c)$

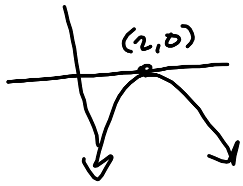
x^2



$-x^2$

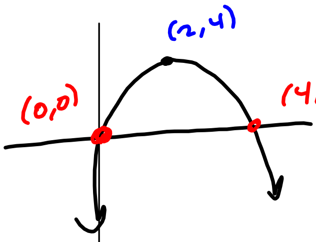


$-(x-2)^2$



$-(x-2)^2+4$

up 4!



$(4,0) \rightsquigarrow$ from $-(x-2)^2+4=0$

or $4x-x^2=0$

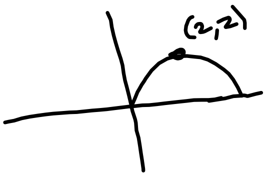
$x(4-x)=0$

$x=0,4$

See §1.3 #3

what about $\sqrt{4x-x^2}$

$$\mathcal{D} = \{x \mid 4x-x^2 \geq 0\} = [0,4]$$



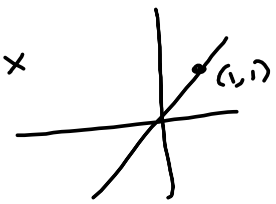
Graphing by shifting and stretching/shrinking/reflecting

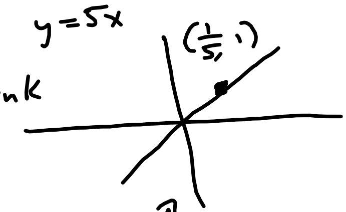
$f(x)$ has (x,y) on it

$f(2x)$ has $(\frac{1}{2}x, y)$

HORIZONTAL SHRINK/stretch

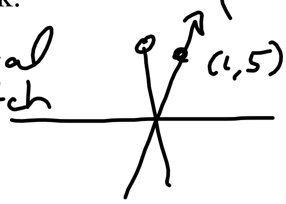
$bf(x)$ has (x, by)





It ain't always easy to get the a's or b's outside the function,

or we could just treat everything like a vertical stetch or shrink.



$g(x) = \sqrt{5x}$ $\quad x \mapsto \frac{1}{5}x$ HOR.

$= \sqrt{5}\sqrt{x}$ $\quad y \mapsto \sqrt{5}\, y$ VERT

HOR $f(x-c)$ is a delay

$(x,y) \longmapsto (x+c, y)$ RIGHT c

RIGID TRANSFORMATIONS (Shifts)

$x^2 \rightsquigarrow (x+7)^2$ LEFT 7

$x^2 \rightsquigarrow (x-3)^2$ RIGHT 3

VER $f(x) + d$ up d units

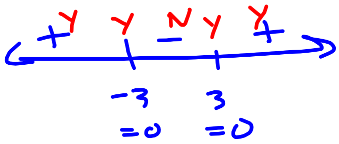
14. 0/8 points

Find each of the following functions and state their domains. (Enter the domains in interval notatic

$f(x) = \sqrt{4 - x}, \quad g(x) = \sqrt{x^2 - 9}$

Need
$4 - x \geq 0$
$4 \geq x$
$x \leq 4$
$(-\infty, 4]$

$x^2 - 9 = (x-3)(x+3) \geq 0$



$(-\infty, -3] \cup [3, \infty)$

$f+g = f(x) + g(x) = \sqrt{4-x} + \sqrt{x^2-9}$

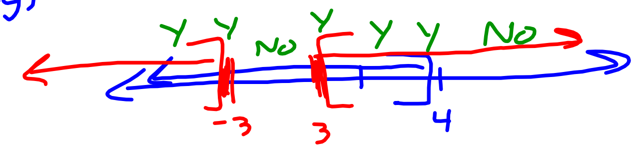
$f-g$

$fg = f(x)\,g(x) = \sqrt{4-x}\sqrt{x^2-9}$

$\dfrac{f}{g} = \dfrac{f(x)}{g(x)} = \dfrac{\sqrt{4-x}}{\sqrt{x^2-9}}$

AND

$\mathcal{D}(f+g) = \mathcal{D}(f) \cap \mathcal{D}(g) = (-\infty, 4] \cap (-\infty, -3] \cup [3, \infty)$



$\mathcal{D}(fg) = \mathcal{D}(f+g) = \mathcal{D}(f-g) \quad (-\infty, -3] \cup [3, 4]$

$\mathcal{D}\left(\frac{f}{g}\right) = \{x \mid x \in \mathcal{D}(f) \cap \mathcal{D}(g) \text{ AND } g(x) \neq 0\}$

So throw out where $g(x) = 0$, i.e., $x = \pm 3$,

so $\boxed{\mathcal{D}\left(\frac{f}{g}\right) = (-\infty, -3) \cup (3, 4]}$