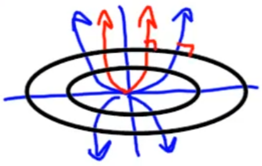
Two curves are **orthogonal** if their tangent lines are perpendicular at each point of intersection. Are the given families of curves **orthogonal trajectories** of each other? That is, is every curve in one family orthogonal to every curve in the other family?

16 $y = cx^2, \; x^2 + 2y^2 = k$

- ○ Yes, the given families of curves are orthogonal trajectories.
- ○ No, the given families of curves are not orthogonal trajectories.



The idea

Sketch both families of curves on the same axes.

I'm not getting negative reciprocals for the slopes.

$y = cx^2 \Longrightarrow y' = 2cx$

$x^2 + 2y^2 = K$

$2x + 4yy' = 0$

$y' = -\frac{2x}{4y} = -\frac{x}{2y} = \pm\frac{-x}{\sqrt{\frac{K-x^2}{2}}}$ NO

$\cancel{2y^2 = K - x^2}$

$\cancel{y^2 = \frac{K-x^2}{2}}$

$\cancel{y = \pm\sqrt{\frac{K-x^2}{2}}}$

$y' = -\frac{2x}{4y}$ & $y = cx^2$

$\Rightarrow y' = -\frac{2x}{4(cx^2)} = \frac{-1}{2cx}$

@ the intersection of $x^2+2y^2=K$ & $y=cx^2$

if $y = cx^2$, then $y' = 2cx$

Hmmmmm $2cx = \frac{-1}{-\frac{1}{2cx}}$

Slope of $y=cx^2$ is $\frac{-1}{\text{slope of } x^2+2y^2=K}$

The WebAssign "Watch It" is much better than my treatment.

I left out looking for the slope when the curves intersect. They occur when $y = cx^2$ Substitute cx^2 for y in the equation $y' = -x/(2y) = -x/(2(cx^2)) =$

$-1/(2cx)$ = the negative reciprocal of $2cx$! That means they're perpendicular whenever they intersect, which is a crazy cool result.

2.8 Related Rates

2. (a) If A is the area of a circle with radius r and the circle expands as time passes, find dA/dt in terms of dr/dt.

(b) Suppose oil spills from a ruptured tanker and spreads in a circular pattern. If the radius of the oil spill increases at a constant rate of 1 m/s, how fast is the area of the spill increasing when the radius is 30 m?

(a) A = Area of circle w/ radius $r \Rightarrow$

$A = A(r) = \pi r^2$

Assume $r = r(t)$ is a function of time. $\Rightarrow A = A(r(t))$

We find dA/dt. $\Rightarrow A'(r(t)) = A'(r(t))\, r'(t)$

$= \frac{dA}{dr} \cdot \frac{dr}{dt}$

So, $\frac{dA}{dt} = \frac{dA}{dr} \cdot \frac{dr}{dt} = \boxed{2\pi r \cdot \frac{dr}{dt} = \frac{dA}{dt} \ \frac{m^2}{s}}$

(b) $\left.\frac{dA}{dt}\right|_{r=30,\ \frac{dr}{dt}=1} = 2\pi (30)(1) = \boxed{60\pi \ \frac{m^2}{sec^2} = \left.\frac{dA}{dt}\right|_{r=30,\ \frac{dr}{dt}=1}}$

Given: $\frac{dr}{dt} = 1$!

Falling Body, by Newton

$h(t)$ = height of a body in free fall, where

t = time, in seconds. Then

$h(t) = -\frac{1}{2}gt^2 + v_0 t + h_0$, where

v_0 = initial velocity and

h_0 = initial height

$g = -\frac{32\,ft}{s^2} = -9.8\,\frac{m}{s^2}$ (on Earth)

These are both the wrong sign, given the $-\frac{1}{2}gt^2$ in the model! The contribution of the gravity term must be negative. Often they are given as $-32\,ft/s^2$ & $-9.8\,m/s^2$, but in that case $+\frac{1}{2}gt^2 + v_0 t + h_0$ is the model!

$v(t) = h'(t) =$ velocity $= \frac{dh}{dt}$ = "speed" but speed has no direction.

up is +

Down is −

A particle moves according to a law of motion $s = f(t)$, $t \geq 0$, where t is measured in seconds and s in feet.

$f(t) = 0.01t^4 - 0.06t^3$

2.8 #1

(a) Find the velocity at time t (in ft/s).

(b) What is the velocity after 1 second(s)?

(c) When is the particle at rest?

(d) When is the particle moving in the positive direction? (Enter your answer using interval notation.)

(e) Find the total distance traveled during the first 11 seconds. (Round your answer to two decimal places.)

(f) Find the acceleration at time t (in ft/s^2).

Find the acceleration after 1 second(s).

(g) Graph the position, velocity, and acceleration functions for the first 11 seconds.

(h) When, for $0 \leq t < \infty$, is the particle speeding up? (Enter your answer using interval notation.)

When, for $0 \leq t < \infty$, is it slowing down? (Enter your answer using interval notation.)

(a) Find the velocity at time t (in ft/s). $f(t) = 0.01t^4 - 0.06t^3$

Want $v(t) = \boxed{f'(t) = .04t^3 - .18t^2 \ \frac{ft}{s}}$

(b) What is the velocity after 1 second(s)?

$v(1) = .04(1)^3 - .18(1)^2 = .04 - .18 = \boxed{-.12 \ ft/s = v(1)}$

(c) When is the particle at rest?

Want $v(t) = .04t^3 - .18t^2 = 0 \implies$

$4t^3 - 18t^2 = 2(2t^3 - 9t^2) = 2t^2(2t-9) = 0$

$\implies \boxed{t = 0, \frac{9}{2} \text{ sec}}$

(d) When is the particle moving in the positive direction? (Enter your answer using interval notation.)

Want $v(t) > 0$: $.04t^3 - .18t^2 = 0 \implies 2t^3 - 9t^2 > 0$

By (c), $= s'(t)$

N N N N Y

− − +

0 $\frac{9}{2}$

$=0$ $=0$

>0

Velocity Question

$\implies t \in (\frac{9}{2}, \infty)$

(e) Find the total distance traveled during the first 11 seconds. (Round your answer to two decimal places.)

Moving left adds to the distance traveled the same as moving right does. We have to calculate the *absolute value* of the net change over the intervals where velocity is monotone increasing and decreasing, separately.

sign Pattern for $v(t) = s'(t)$

− +

0 $\frac{9}{2}$

f'

$f'(t)$

$(0,0)$ $(9/2, f(9/2))$

So we need $|s(\frac{9}{2}) - s(0)| + |s(11) - s(\frac{9}{2})|$

$= |s(\frac{9}{2}) - s(0)| + s(11) - s(\frac{9}{2})$, b/c $s(11) > s(\frac{9}{2})$

$= |s(\frac{9}{2})| + s(11) - s(\frac{9}{2})$

$= -s(\frac{9}{2}) + s(11) - s(\frac{9}{2})$, b/c we know $(0) > (\frac{9}{2})$, i.e., $v(t) < 0$ on $(0, 9/2)$ so it's moving left (negative) on $(0, 9/2)$ by the sign pattern

$= s(11) - 2s(\frac{9}{2})$

```
 Plot1  Plot2  Plot3
\Y1■.01X^4-.06X^
3
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
```

```
WINDOW
 Xmin=-3.3
 Xmax=7
 Xscl=1
 Ymin=-2
 Ymax=3.5
 Yscl=1
↓Xres=■
```

Captures main Features

```
Y1(11)-2Y1(9/2)
         69.28375
■
```

$\approx$ Total Distance $\approx$ 69.28 ft

(f) Find the acceleration at time t (in ft/s^2).

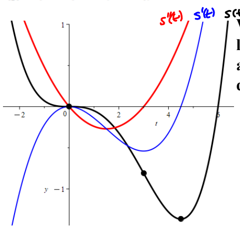
Acceleration $= v'(t) = s''(t) = a(t)$

$= \boxed{.12t^2 - .36t = s''(t)}$

Find the acceleration after 1 second(s).

$s''(1) = .12 - .36 = \boxed{-.24 \ ft/s^2 = s''(1)}$

(g) Graph the position, velocity, and acceleration functions for the first 11 seconds.



We're not interested in anything to the left of the *y*-axis, but I included it. We are interested in going up to *t* = 11, but I didn't.

(h) When, for $0 \leq t < \infty$, is the particle speeding up? (Enter your answer using interval notation.)

When, for $0 \leq t < \infty$, is it slowing down? (Enter your answer using interval notation.)

$s'(t) = .04t^3 - .18t^2 \stackrel{\text{SET}}{=} 0 \implies$

− ... +

$= 4t^3 - 18t^2 = 0 \implies$

$2t^2(2t-9) = 0 \implies$

$t = 0, \frac{9}{2}$

$s''(t) = .12t^2 - .36t \stackrel{\text{SET}}{=} 0 \implies$

$12t^2 - 36t = 0 \implies$

$12t(t-3) = 0 \implies$

$t = 0, 3$

Sign Patterns for s' and s'' :

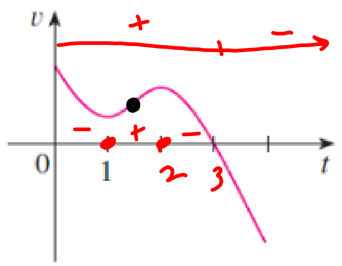
$s'(t)$: − | 0 | − | $\frac{9}{2}$ | +

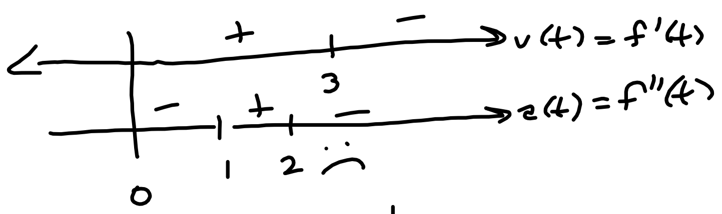
$s''(t)$: + | 0 | − | 3 | +

The particle is speeding up when the acceleration is in the same direction as the velocity. *That's* what I wasn't grokking, yesterday. So it's speeding up when

$t \in (0, 3) \cup (9/2, \infty)$

(b)





Speeding up

$(1,2) \cup (3,4)$

When is the particle in figure (b) speeding up? (Enter your answer using interval notation.)

$(1,2) \cup (\ )$ ✗

When is the particle in figure (b) slowing down? (Enter your answer using interval notation.)

$(0,1) \cup (2,3)$ ✓

If a ball is thrown vertically upward with a velocity of 144 ft/s, then its height after t seconds is $s = 144t - 16t^2$.

(a) What is the maximum height reached by the ball?

[ ] ft

(b) What is the velocity of the ball when it is 320 ft above the ground on its way up? (Consider up to be the positive direction

[ ] ft/s

What is the velocity of the ball when it is 320 ft above the ground on its way down?

[ ] ft/s

$s(t)$ $-16t^2+144t$

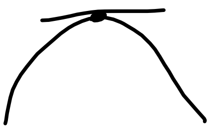
Max height

$s'(t) = -32t + 144 \overset{SET}{=} 0$

$\Rightarrow -32t = -144$

$t = \frac{144}{32} = \boxed{\frac{9}{2} = t}$

$s(\frac{9}{2})$ is max height



(b) Algebra:
Set $s(t) = 320$
& plug t into $v(t)$

$-16t^2 + 144t = 320$

$-16t^2 + 144t - 320 = 0$

$-16(t^2 - 9t + 20) = 0$

$(t-5)(t-4) = 0$

$\Rightarrow t = 4, 5$

UP: $v(4)$

DOWN $v(5)$