Continuing our Review of Chapter 3 and likely Test Questions.

Deadline for Test 3 is Midnight, Wednesday night.

I'll open up ZOOM on Wednesday, but if there are no questions, I'm going to set everyone loose to take the test.

Miscellaneous Mean Value Theorem questions. Section 3.2

9. 0/2 points

Suppose that $4 \le f'(x) \le 5$ for all values of x. What are the minimum and maximum possible values of $f(4) - f(1)$?

$1 \le f'(x) \le 2$

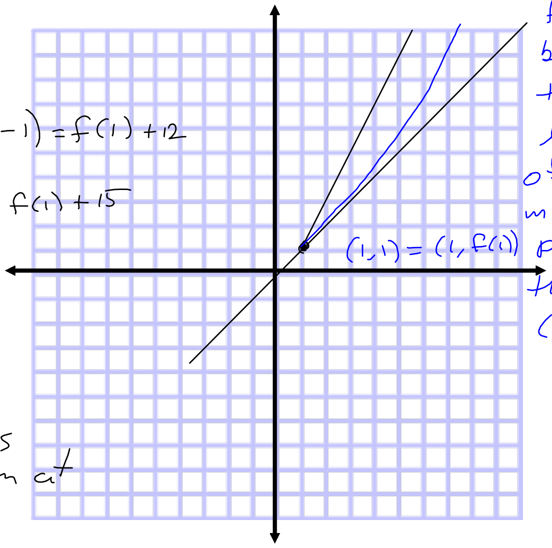
$f(4) \ge f(1) + 4(4-1) = f(1) + 12$

$f(4) \le f(1) + 5(4-1) = f(1) + 15$

$\Rightarrow f(4) - f(1) \ge 12$ &

$\Rightarrow f(4) - f(1) \le 15$

Bounds on f' gives bounds on f, if $f(x)$ is known at one point.



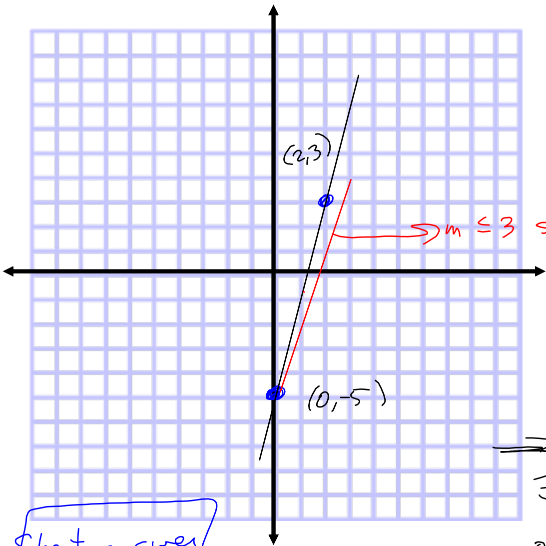
f lives between the lines of slope $m=1$ & $m=2$ passing thru $(1, f(1))$

10. 0/1 points

Does there exist a function f such that $f(0) = -5$, $f(2) = 3$, and $f'(x) \le 3$ for all x?

$(0,-5)$ & $(2,3)$

$$m_{AVG} = m = \frac{-5-3}{0-2} = \frac{-8}{-2} = 4$$



$m_{AVG} = m_{sec} = 4$

$\Rightarrow m \le 3$ says f lives under this line, & it doesn't!

Slick answer:

f is differentiable $\forall x$

" " cont^s " "

$\Rightarrow$ MVT holds $\Rightarrow$

$\exists\ c \in (0,2) \ni f'(c) = 4 = m_{AVG}$.

$\therefore f'(x) \le 3\ \forall x$ is impossible.

$\boxed{\text{Short answer}}$

MVT says $\exists\ c \in (0,2)$

$\ni f'(c) = m_{avg} = 4 > 3$ so

$f'(x) \le 3$ doesn't hold.

5. 0/7 points SCalc8 3.3.009. [3353668]

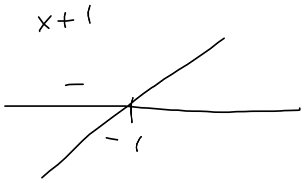
Consider the equation below. (If an answer does not exist, enter DNE.)

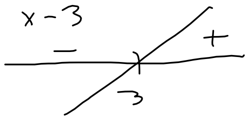
$f(x) = x^3 - 3x^2 - 9x + 7$

Intervals of increase, decrease, and concavity. Want to show how the book does it, if people want. Then show how I'd do it.

(a) $f(x) = x^3 - 3x^2 - 9x + 7 \Rightarrow f'(x) = 3x^2 - 6x - 9 = 3(x^2 - 2x - 3) = 3(x + 1)(x - 3).$

Interval	$x + 1$	$x - 3$	$f'(x)$	f
$x < -1$	−	−	+	increasing on $(-\infty, -1)$
$-1 < x < 3$	+	−	−	decreasing on $(-1, 3)$
$x > 3$	+	+	+	increasing on $(3, \infty)$

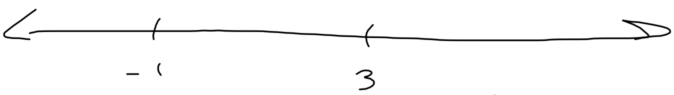




$f(x) = x^3 - 3x^2 - 9x + 7 \Rightarrow$

$f'(x) = 3x^2 - 6x - 9 = 3(x^2 - 2x - 3) \overset{\text{SET}}{=} 0 \Rightarrow$

$(x-3)(x+1) = 0 \Rightarrow x \in \{-1, 3\}$



$(-\infty, -1)$ $(-1, 3)$ $(3, \infty)$

$x < -1$ $-1 < x < 3$ $3 < x$

$(x+1)(x-3)$

End Behavior $3x^2$

$(x-3)(x+1)$

J

MAT 121

Question :

11. (2 pts) What is the domain of $W(x)=\sqrt{(x-3)^2(x+3)^3(x+2)^4(x-8)}$?

$= \sqrt{\text{STUFF}}$

12. (2 pts) What is the domain of $K(x)=\sqrt{\dfrac{(x-3)^2(x-8)}{(x+3)^3(x+2)^4}}$?

$\text{STUFF} = (x-3)^2(x+3)^3(x+2)^4(x-8)^1 = x^{2+3+4+1} + \cdots = x^{10} + \cdots$

#11 Need STUFF ≥ 0.

x^{10}

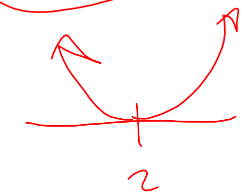
$+ \quad - \quad - \quad - \quad +$

$-3 \quad -2 \quad 3 \quad 8$

$=0 \quad =0 \quad =0 \quad =0$

≥ 0

want ≥ 0



$\Rightarrow \mathcal{D} = (-\infty,-3] \cup \{-2, 3\} \cup [8, \infty)$

$\ln\left((x+5)^3(x-7)^6(x+2)(x-8)^9\right)$ what's Domain?

$= \ln(\text{stuff})$. Need stuff > 0 $\quad$ stuff $= x^{19} + \cdots$

x^{19}

$- \quad + \quad - \quad - \quad +$

$-5 \quad -2 \quad 7 \quad 8$

> 0

$\mathcal{D} = (-5,-2) \cup (8, \infty)$

$f(x) = (x+1)^7 - 7x - 2$

Find int's of inc/dec. & concavity.

$f(x) = (x+1)^7 - 7x - 2 \implies$

$f'(x) = 7(x+1)^6 - 7 \overset{\text{SET}}{=} 0 \implies$

$(x+1)^6 - 1 = 0$

$(x+1)^6 = 1$

$\sqrt[6]{(x+1)^6} = \sqrt[6]{1}$

$|x+1| = 1$

$x+1 = \pm 1$

$x = -1 \pm 1 \to -2, \; 0$

$(x+1)^6 = \left((x+1)^2\right)^3$ OR $\left((x+1)^3\right)^2$

$\left((x+1)^3\right)^2 - 1^2 = \left((x+1)^3 - 1\right)\left((x+1)^3 + 1\right)$

$(x+1)^3 - 1 = (x+1)^3 - 1^3 = a^3 - b^3$

$= \underbrace{\left((x+1) - 1\right)}_{x=0}\left((x+1)^2 + 1(x+1) + 1^2\right)$

$(x+1)^3 + 1 = (x+1)^3 + 1^3$

$= \underbrace{\left((x+1)+1\right)}_{x=-2}\left((x+1)^2 - (x+1) + 1^2\right)$

$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

$a^3 + b^3 = (a+b)(a^2 - ab + b)$

$7(x+1)^6 - 7x = f'(x)$

f': $+$ on $(-\infty, -2)$ ↗, $-$ on $(-2, 0)$ ↘, $+$ on $(0, \infty)$ ↗

$f'(x) = 7x^6 + \text{smaller stuff}$

$\nwarrow \ldots \nearrow$

The quick "power of the factor" is a bit harder to apply. When in doubt, use a test value inside the interval.

$x = -1: \; 7(-1+1) - 7 = 0 - 7 < 0 \quad -$

Inc: $(-\infty, -2) \cup (0, \infty)$

Dec: $(-2, 0)$

$f'(x) = 7(x+1)^6 - 7 \implies$

$f''(x) = 42(x+1)^5 \overset{\text{SET}}{=} 0 \implies x = -1$

f'': $-$ on $(-\infty, -1)$ ⌢, $+$ on $(-1, \infty)$ ⌣

$42x^5 + \ldots$

$- \; \swarrow \ldots \nearrow \; +$

c-up: $(-1, \infty)$

c-down: $(-\infty, -1)$

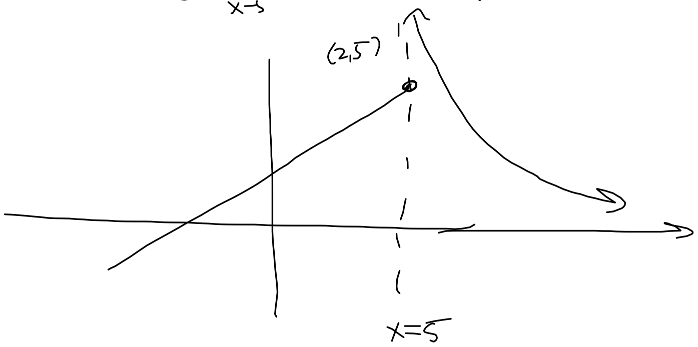
1. 0/3 points

(a) Can the graph of $y = f(x)$ intersect a vertical asymptote?

- Yes
- No

, But it takes a piecewise function to pull it off! Otherwise, no!

$$f(x) = \begin{cases} x+2 & \text{if } x \le 5 \\ \frac{1}{x-5} & \text{if } x > 5 \end{cases}$$



(b) How many horizontal asymptotes can the graph of $y = f(x)$ have? (Select all that apply.)

- [ ] 0
- [ ] 1
- [ ] 2

$$f(x) = \begin{cases} \frac{1}{x} & \text{if } x < 0 \\ \frac{2x+3}{3x+2} & \text{if } x \geq 0 \end{cases}$$

$\left.\begin{array}{l} y = 0 \\ y = \frac{2}{3} \end{array}\right\}$ 2 H.A.

Takes a piecewise, again.

0 or 1 in 99.99% of cases.

7. 0/1 points

Find the limit, if it exists. (If an answer does not exist, enter DNE.)

$$\lim_{t \to \infty} \frac{\sqrt{t} + t^2}{8t - t^2} = \lim_{t \to \infty} \frac{t^2 + \cdots}{-t^2 + \cdots} = -1$$

8. 0/1 points

Find the limit, if it exists. (If an answer does not exist, enter DNE.)

$$\lim_{x \to \infty} \frac{x^5}{\sqrt{x^{10} + 9}}$$

$$\frac{x^5}{\sqrt{x^{10}+9}} = \frac{x^5}{\sqrt{x^{10}\left(1 + \frac{9}{x^{10}}\right)}} = \frac{x^5}{|x^5| \sqrt{1 + \frac{9}{x^{10}}}} = \frac{x^5}{x^5 \sqrt{1 + \frac{9}{x^{10}}}}$$

$\left(x > 0 \text{ from } x \to +\infty \right)$

$$= \frac{1}{\sqrt{1 + \frac{9}{x^{10}}}} \xrightarrow{x \to \infty} 1$$

$\frac{9}{x^{10}} \to 0$

what about $\lim_{x \to -\infty} \frac{x^5}{\sqrt{x^{10}+9}}$

$$\frac{x^5}{\sqrt{x^{10}+9}} = \cdots \frac{x^5}{|x^5| \sqrt{1 + \frac{9}{x^{10}}}} = \frac{x^5}{-x^5 \sqrt{1 + \frac{9}{x^{10}}}} = \frac{-1}{\sqrt{1 + \frac{9}{x^{10}}}}$$

b/c $x < 0$ in $x \to -\infty$.

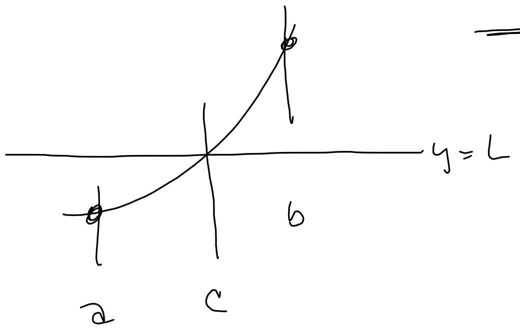
$\frac{9}{x^{10}} \to 0$

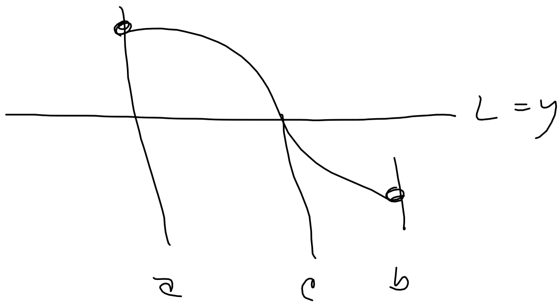
$$\xrightarrow{x \to -\infty} -1.$$

IVT : f cnt^s : $a < b$ & $f(a) < L < f(b)$ OR

Intermediate value Theorem. $f(a) > L > f(b)$

$\Longrightarrow \exists\, c \in (a,b) \ni f(c) = L$





EVT : f cnt^s on $[a,b]$

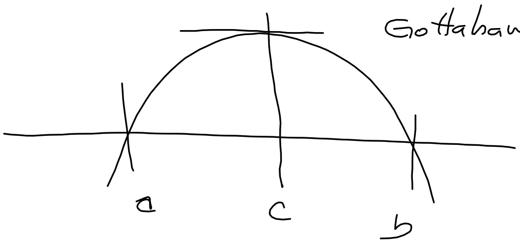
$\Longrightarrow$ f achieves an absolute max & min in $[a,b]$

continuous on closed interval stuff

Rolle's : f cnt^s $[a,b]$ & f dif^{ble} on (a,b) & $f(a)=f(b)$

$\Longrightarrow \exists\, c \in (a,b) \ni f'(c) = 0$

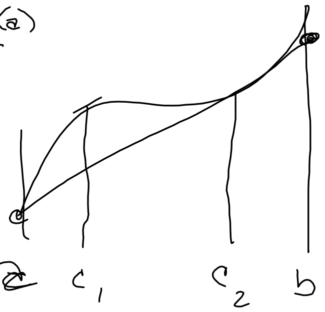
Gotta have a flat spot



MVT

Mean Value Theorem (for derivatives).

f cnt^s on $[a,b]$ & difb^l on (a,b) $\Rightarrow \exists$
$c \in (a,b) \ni f'(c) = \dfrac{f(b)-f(a)}{b-a}$



MVT says it's there, but doesn't tell you how to find it.

$f(x) = \dfrac{x+2}{x-3}$. Find $c \in (4,6)$ that satisfies the conclusion of MVT.

(a) Does MVT apply?

f is cnt^s and difb^l on its domain!

$\mathcal{D}(f) = \mathbb{R} \setminus \{3\} \supset [4,6] \supset (4,6)$

cnt^s ✓ difb^l ✓

Yes.

So $f'(x) = \dfrac{1(x-3)-(x+2)(1)}{(x-3)^2} = \dfrac{x-3-x-2}{(x-3)^2} = \dfrac{-5}{(x-3)^2}$

& $m_{sec} = m_{avg} = \dfrac{f(6)-f(4)}{6-4} = \dfrac{\frac{8}{3}-\frac{6}{1}}{2} = \dfrac{\frac{8}{3}-\frac{18}{3}}{2} = \dfrac{\frac{-10}{3}}{2} = -\frac{5}{3}$

$= m_{avg} = -\frac{5}{3} \overset{SET}{=} f'(x) \Rightarrow$

$\dfrac{-5}{(x-3)^2} \overset{SET}{=} -\dfrac{5}{3}$

$-15 = -5(x-3)^2$

$3 = (x-3)^2 = 3$

$x-3 = \pm\sqrt{3}$

$x = 3 \pm \sqrt{3} \longrightarrow \boxed{c = 3+\sqrt{3}}$

Check: $\dfrac{-5}{(3+\sqrt{3}-3)^2} = \dfrac{-5}{(\sqrt{3})^2} = -\dfrac{5}{3} = m_{AVG}$! ✓