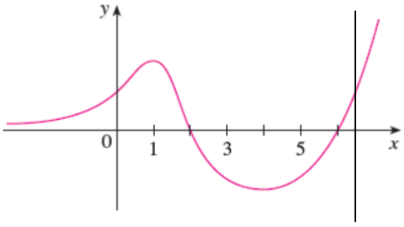
2. 0/5 points

For each initial approximation, determine graphically what happens if Newton's method is used for the function whose graph is shown.



$x_1 = 1$! Fails!

$x_1 = 2$ looks like a zero!

$x_1 = 0$ Fails!

$x_1 = 3$ appears to work

$x_1 = 4$ Horizontal tangent fail

$x_1 = 5$ succeeds

4. 0/1 points

Use Newton's method with the specified initial approximation x_1 to find x_3, the third approximation to the root of the given equation. (Round your answer tc

$$\frac{2}{x} - x^2 + 1 = 0, \quad x_1 = 2$$

$x_3 =$ ✗ 1.5215

$$f(x) = 2x^{-1} - x^2 + 1 = \frac{2}{x} - x^2 + 1$$

$$f'(x) = -2x^{-2} - 2x = \frac{-2}{x^2} - 2x$$

$$f(2) = \frac{2}{2} - 2^2 + 1 = -2$$

$$f'(2) = \frac{-2}{2^2} - 2(2) = -\frac{1}{2} - 4 = -\frac{9}{2}$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= 2 - \frac{-2}{-\frac{9}{2}} = 2 - 2\left(\frac{2}{9}\right) = 2 - \frac{4}{9} = \frac{18-4}{9} = \boxed{\frac{14}{9} = x_2}$$

$$f'(x) = -\frac{2}{x^2} - 2x$$

$$x_2 = 2 - \frac{-2}{\left(-\frac{9}{2}\right)} = 2 - 2\left(\frac{2}{9}\right) = 2 - \frac{4}{9} = \frac{14}{9} \checkmark$$

See video for how this was built using graphing calculator!

```
Plot1 Plot2 Plot3
\Y1=2/X-X²+1
\Y2=-2/X²-2X
\Y3=X-Y1(X)/Y2(X
)
\Y4=
\Y5=
\Y6=
```

```
          1.521515181
Y3(2)
          1.555555556
Y3(Ans)
          1.521515181
Y3(Ans)
          1.521379709
```

The above is a TI-83/TI-84 implementation of this recursion. See Wednesday, 3/24 notes for the recursion with a spreadsheet.

Analyze

$$f(x) = x|x| = \begin{cases} x \cdot x & \text{if } x \geq 0 \\ x \cdot (-x) & \text{if } x < 0 \end{cases} = \begin{cases} x^2 & \text{if } x \geq 0 \\ -x^2 & \text{if } x < 0 \end{cases}$$

You can have a smooth function with an inflection point, even if $f'' \nexists$!

$$f'(x) = \begin{cases} 2x & \text{if } x \geq 0 \\ -2x & \text{if } x < 0 \end{cases}$$

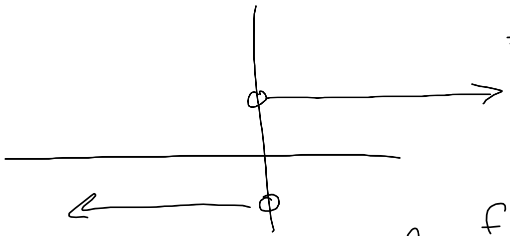
what about @ $x=0$?

$$\lim_{h \to 0^-} \frac{f(0+h) - f(0)}{h} = \lim_{h \to 0} \frac{f(h)}{h} = \lim_{h \to 0} \frac{-h^2}{h} = \lim_{h \to 0} -h = 0$$

$$\lim_{h \to 0^+} \frac{f(0+h) - f(0)}{h} = \dots = 0$$

$$f''(x) = \begin{cases} 2 & \text{if } x > 0 \\ -2 & \text{if } x < 0 \end{cases}$$

@ $x=0$?

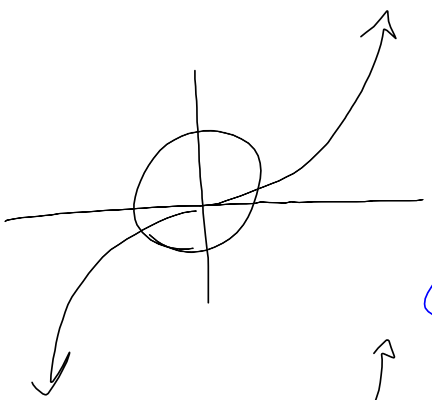


Can we do $f''(0)$?

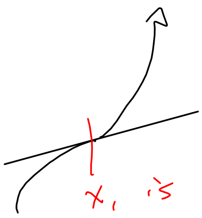
$$\lim_{h \to 0^-} \frac{f'(0+h) - f'(0)}{h} = \lim_{h \to 0^-} \frac{f'(h) - f'(0)}{h} = \lim_{h \to 0^-} \frac{-2h}{h} = -2$$

$$\lim_{h \to 0^+} \frac{f'(0+h) - f'(0)}{h} = \lim_{h \to 0} \frac{f'(h)}{h} = \frac{2h}{h} = 2$$

$-2 \neq 2 \therefore f''(0) \nexists$



Inflection point is where the graph changes concavity, i.e., where the tangent line @ x_1 crosses the graph @ x_1



x_1 is inflection point!

Find the most general antiderivative of the function.
antiderivative.)

$x \cdot x^{\frac{1}{2}} = x^{3/2}$

Power Rule for $n \neq 1$

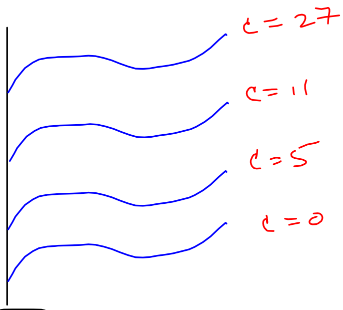
$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$$

$$f(x) = \sqrt[9]{x^2} + x\sqrt{x}$$

$$= x^{\frac{2}{9}} + x^{\frac{3}{2}}$$

$$\Rightarrow \int f(x)\,dx = \frac{x^{\frac{11}{9}}}{\frac{11}{9}} + \frac{2}{5}x^{\frac{5}{2}} + C$$

$$\boxed{= \frac{9}{11}x^{\frac{11}{9}} + \frac{2}{5}x^{\frac{5}{2}} + C}$$



$F(x)$

Find the PARTICULAR function $F(x)$ with $f(x)$ as its derivative that passes thru $(10,5)$

want $F(10) = 5$

we know $F(x) = \frac{9}{11}$ thing $\Rightarrow$

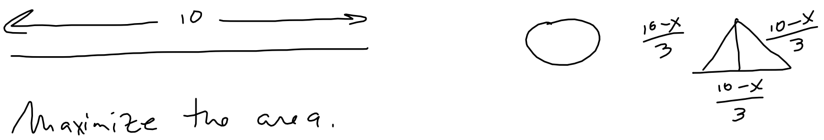
$$F(10) = \frac{9}{11}(10)^{11/9} + \frac{2}{5}(10)^{\frac{5}{2}} + C = 5 \Rightarrow$$

$$\boxed{C = 5 - \frac{9}{11}(10)^{11/9} - \frac{2}{5}(10)^{\frac{5}{2}}}$$

$$\boxed{F(x) = \frac{9}{11}x^{11/9} + \frac{2}{5}x^{\frac{5}{2}} + C}$$

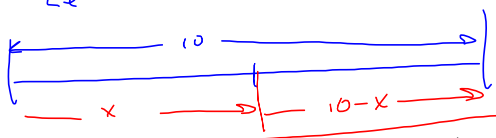
§3.7 Application.

Wire of 10 inches is going to be cut & the 2 pieces formed into a circle & an equilateral triangle. Find the length of the cut that will maximize the enclosed area.



Maximize the area.

Let x = the circumference of the circle (in inches)



$$\text{area} = \pi r^2 + \frac{1}{2}\left(\frac{10-x}{3}\right)\cdot\frac{\sqrt{3}}{2}\left(\frac{10-x}{3}\right)$$

$$\frac{h}{\frac{10-x}{3}} = \sin 60^\circ$$

$$h = (\sin 60^\circ)\left(\frac{10-x}{3}\right)$$

Need r(x):

circumference $= 2\pi r = x$

$r = \frac{x}{2\pi}$

$$\text{Area} = \pi\left(\frac{x}{2\pi}\right)^2 + \frac{1}{2}(10-x)^2\left(\frac{1}{9}\right)\frac{\sqrt{3}}{2} = A(x)$$

$$= \frac{x^2}{4\pi} + \frac{\sqrt{3}}{36}(x-10)^2$$

$$A'(x) = \frac{2}{4\pi}x + \frac{\sqrt{3}}{36}(2(x-10)) = \frac{2}{4\pi}x + \frac{\sqrt{3}}{18}x - \frac{\sqrt{3}}{18}\cdot 10$$

$$= \left(\frac{2}{4\pi} + \frac{\sqrt{3}}{18}\right)x - \frac{5\sqrt{3}}{9} \overset{SET}{=} 0$$

will give us the minimum.

$$x = \frac{\frac{5\sqrt{3}}{9}}{\frac{1}{2\pi} + \frac{\sqrt{3}}{18}} = \frac{5\sqrt{3}}{9\left(\frac{1}{2\pi} + \frac{\sqrt{3}}{18}\right)}$$ minimizes it.

TO MAXIMIZE area, Don't cut! Make a circle!

Either 10" or 0", depending on how you define x

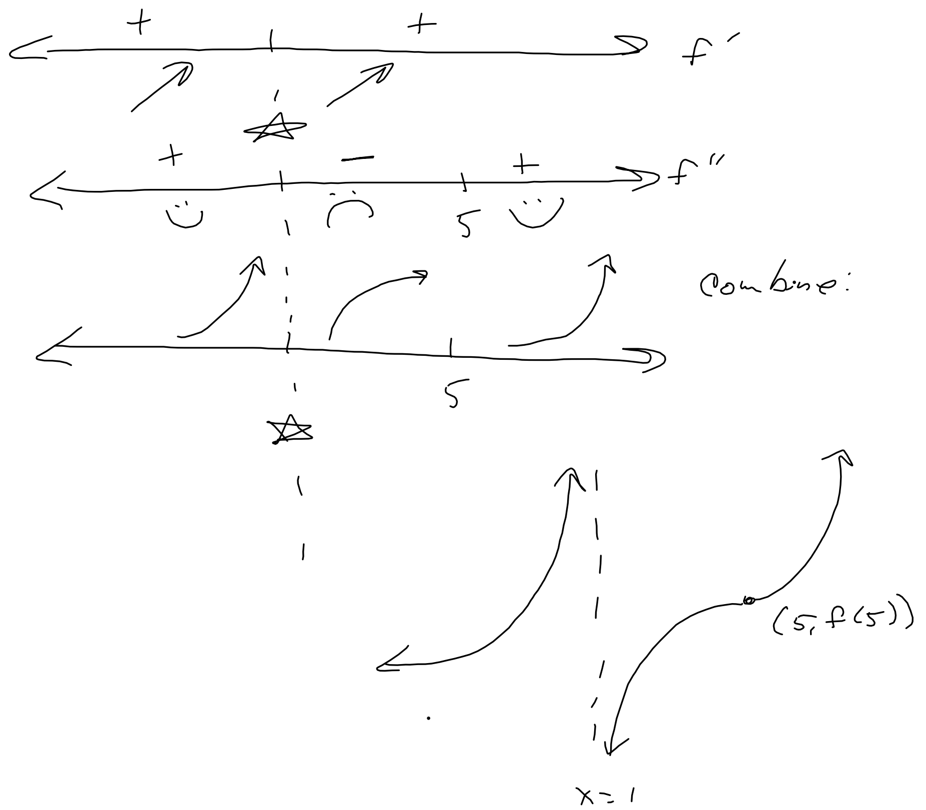
x = perimeter of circle? x = 10

x = " " triangle? x = 0

11. 0/1 points

Sketch the graph of a function that satisfies all of the given conditions.

$f'(x) > 0$ for all $x \neq 1$,
vertical asymptote $x = 1$,
$f''(x) > 0$ if $x < 1$ or $x > 5$,
$f''(x) < 0$ if $1 < x < 5$



$f(x) = 5x^3 - 3x^5$

(a) Inc & dec

(b) Local Ext

(c) IPs

(d) C-up, C-down

$f(x) = 0 \Rightarrow$

$x^3(5 - 3x^2)$

$x = 0$ or $3x^2 = 5$

$x^2 = \frac{5}{3}$

$x = \pm\sqrt{\frac{5}{3}} = \pm\frac{\sqrt{15}}{3}$

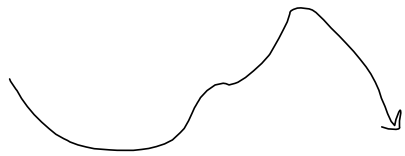
$D = \mathbb{R}$

(a) $f'(x) = 15x^2 - 15x^4 = 15x^2(1 - x^2) \overset{SET}{=} 0 \Rightarrow$

$x = \pm 1, 0$

$x^2(1-x)(1+x)$

$-$ -1 $+$ 0 $+$ 1 $-$



Inc: $\boxed{(-1, 0) \cup (0, 1)}$

dec: $\boxed{(-\infty, -1) \cup (1, \infty)}$

(b) Local min: $(-1, f(-1)) = (-1, -2)$

Local max $(1, f(1)) = (1, 2)$

$f(-1) = 5(-1) - 3(-1) = -2$

(c) $15x^2 - 15x^4 = f'(x) \Rightarrow$

$f''(x) = \boxed{30x - 60x^3 = 0} \Rightarrow$

$30x(1 - 20x^2) = 0 \Rightarrow$

$x = 0$ or $x = \pm\sqrt{\frac{1}{20}} = \pm\frac{1}{2\sqrt{5}} = \pm\frac{\sqrt{5}}{10}$

Plug those in to get y values.

But this is the main ide

f'': $+$ $-\frac{\sqrt{5}}{10}$ $-$ 0 $+$ $\frac{\sqrt{5}}{10}$ $-$

C-up: $(-\infty, -\frac{\sqrt{5}}{10}) \cup (0, \frac{\sqrt{5}}{10})$

C-Down: $(-\frac{\sqrt{5}}{10}, 0) \cup (\frac{\sqrt{5}}{10}, \infty)$