Quick Survey of What Remains in Chapter 3:

3.5 Graphing Summary - Field Questions

3.6 Graphing with Calculus and Calculators - This is what we've BEEN doing. Will field questions.

3.7 Optimization Problems - Some examples from the exercises

3.8 Newton's Method - The basic theory. Some Spreadsheet Stuff?

3.9 Antiderivatives - Saying things about where something is, when you know how it's moving.

Power Rule. $\int x^{n+1} dx = \frac{x^{n+1}}{n+1} + C$

More antiderivatives:

$\frac{d}{dx}\sin(x) = \cos(x) \Longrightarrow \int \cos(x)\,dx = \sin(x) + C$

$\frac{d}{dx}\cos(x) = -\sin(x) \Longrightarrow \int \sin(x)\,dx = -\cos(x) + C$

$\frac{d}{dx}\tan(x) = \sec^2(x) \Longrightarrow \int \sec^2(x)\,dx = \tan(x) + C$, but

what about $\int \tan(x)\,dx$? $-\int \frac{-\sin(x)}{\cos(x)}$

OBVIOUSLY $\int \tan(x)\,dx = -\ln|\cos(x)| + C = \ln\left(|\cos(x)|^{-1}\right) + C$

$= \boxed{\ln|\sec(x)| + C}$

(NOT OBVIOUS. WILL TAKE SOME MORE THEORY)

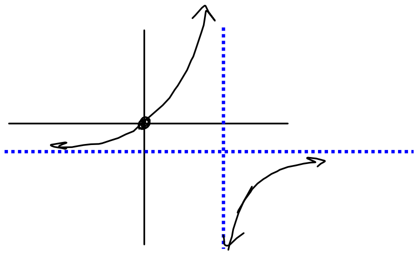
$$\boxed{\int \sec(x)\,dx = \ln|\sec(x) + \tan(x)| + C}$$

S3.5 #6: Always check for holes!

$R(x) = \frac{-x^2+x}{x^2-7x+6} = \frac{-x(x-1)}{(x-6)(x-1)} = -\frac{x}{x-6} = R^*(x)$ — $y=-1$ is H.A.

$-\frac{x}{x-6} \xrightarrow{x\to\pm\infty} -\frac{x}{x} = -1$

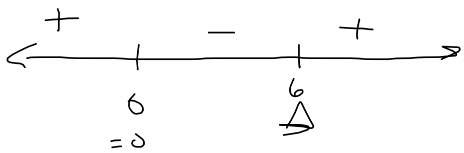
HOLE: $R^*(1) = \frac{-1}{1-6} = \frac{-1}{-5} = \frac{1}{5} \rightsquigarrow$ Hole @ $(1, \frac{1}{5})$



$R(x)=0$ @ $x=0 \to (0,0)$

$R(x)=\text{DNE}$ @ $x=6$ V.A.

$R(x)$ has hole @ $x=1$.



Calculus stuff should confirm this MAT 121 pic. One thing to check:

See if $R^*(x)=1$

$\frac{-x}{x-6} = 1 \Rightarrow$

$\frac{-x}{x-6} = \frac{x-6}{x-6}$

$\frac{-x-(x-6)}{x-6} = 0$

$\frac{-x-x+6}{x-6} = 0$

$\frac{-2x+6}{x-6} = 0$

$-2x = -6$

See if $R^*(x) = -1$!

$\frac{-x}{x-6} = -1$

$\frac{-x}{x-6} = \frac{-x+6}{x-6}$

$\frac{-x+x-6}{x-6} = \frac{-6}{x-6} = 0$

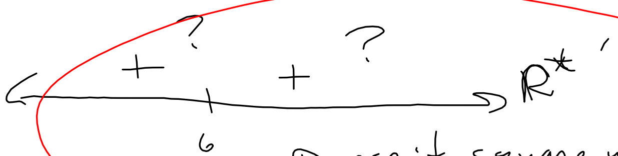
Never! $\frac{A}{B}=0$ iff $A=0$ &

$-6 \neq 0$

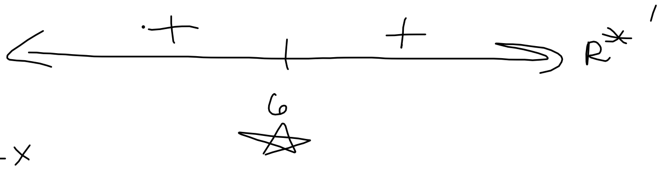
$x = \frac{-6}{-2} = 3$ so it Does NOT cross, so it MUST have some "wiggle" to it off to the right of $x=3$. Just work with $R^*(x)$.

$$R^* = \frac{-x}{x-6} \Rightarrow R^{*\prime} = \frac{-1(x-6) - (-x)(1)}{(x-6)^2} = \frac{-x+6+x}{(x-6)^2} = \frac{6}{(x-6)^2}$$

No zeros. Blows up @ $x=6$. No wiggle.



Doesn't square with the 121 Pic! wrong!



$$R^* = \frac{-x}{x-6}$$

$$R^{*\prime} = \frac{-1(x-6) - (-x)(1)}{(x-6)^2} = \frac{-x+6+x}{(x-6)^2} = \frac{6}{(x-6)^2} = 6(x-6)^{-2}$$

$$\Rightarrow R^{*\prime\prime} = -12(x-6)^{-3} = \frac{-12}{(x-6)^3}$$

Limit @ infinity

#9 $y = \sqrt{x^2+2x} - x$

$\lim_{x \to +\infty} (y)$:

$$\left(\sqrt{x^2+2x} - x\right)\left(\frac{\sqrt{x^2+2x} + x}{\sqrt{x^2+2x} + x}\right) = \frac{\left(\sqrt{x^2+2x}\right)^2 - x^2}{\quad}$$

($a - b$, $a + b$, $a^2 - b^2$)

$$= \frac{x^2+2x-x^2}{\sqrt{x^2+2x} + x} = \frac{2x}{x\sqrt{1+2/x} + x} = \frac{2x}{x\left(\sqrt{1+\frac{2}{x}} + 1\right)} = \frac{2}{\sqrt{1+\frac{2}{x}} + 1}$$

($\frac{2}{x} \to 0$)

$\xrightarrow{x \to +\infty} \frac{2}{2} = 1$! That's the horizontal asymptote !

$y = 1$

$$\sqrt{x^2\left(1+\frac{2x}{x^2}\right)} \quad \sqrt{x^2\left(1+\frac{2}{x}\right)} = \sqrt{x^2}\sqrt{1+\frac{2}{x}} = |x|\sqrt{1+\frac{2}{x}}$$

$$= x\sqrt{1+\frac{2}{x}}$$

Slant Asymptotes:

$$y = \frac{2x^3+x^2+x+4}{x^2+2x}$$

For slant/oblique Asymptotes, do the long division:

$$\begin{array}{r} 2x - 3 \\ x^2+2x \overline{) \; 2x^3+x^2+x+4} \\ -(2x^3+4x^2) \\ \hline -3x^2 \\ -(-3x^2 \quad) \end{array}$$

$y = 2x - 3$ is O.A.

3.6 #2 Another O.A. This one's quadratic!

$$\frac{x^4 - x^3 - 8}{x^2 - x - 6} = \frac{(x-2)(x+1.477..)(\text{quadratic factor})}{(x-3)(x+2)}$$

-1.477967242

$x^4 - x^3 - 8$ Check for zeros:

$$\begin{array}{r|rrrrr} -2 & 1 & -1 & 0 & 0 & -8 \\ & & -2 & 6 & -12 & \\ \hline & 1 & -3 & 6 & & \end{array}$$

zeros of Numerator?
Resort to technology

$x = 2$:

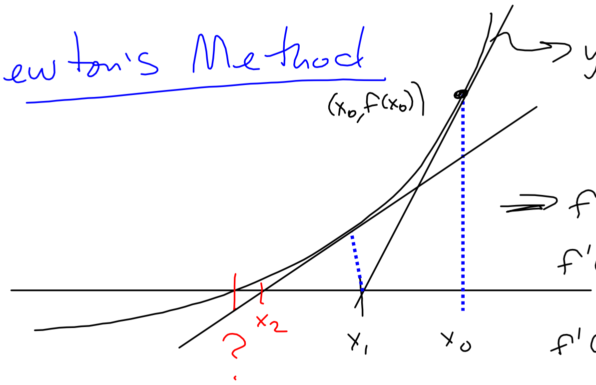
$$\begin{array}{r|rrrrr} 2 & 1 & -1 & 0 & 0 & -8 \\ & & 2 & 2 & 4 & 8 \\ \hline & 1 & 1 & 2 & 4 & 0 \end{array}$$

$x \approx -1.47...$ was also an x-intercept.

$$\begin{array}{r|l} & x^2 + 6 \\ x^2 - x - 6 & x^4 - x^3 + 0x^2 + 0x - 8 \\ & -(x^4 - x^3 - 6x^2) \\ \hline & 6x^2 \sim \\ & -(6x^2 \sim) \\ \hline & \sim \end{array}$$

$y = x^2 + 6$ is O.A.

Newton's Method



$y = f'(x_0)(x - x_0) + f(x_0) \overset{SET}{=} 0$!

x_1 !

$\Rightarrow f'(x_0)(x_1 - x_0) = -f(x_0)$

$f'(x_0)x_1 - f'(x_0)x_0 = -f(x_0)$

$f'(x_0)x_1 = -f(x_0) + f'(x_0)x_0$

$\Rightarrow x_1 = \dfrac{-f(x_0)}{f'(x_0)} + \dfrac{f'(x_0)}{f'(x_0)} x_0$

$x_1 = x_0 - \dfrac{f(x_0)}{f'(x_0)}$

Newton's Method!

Will show a spreadsheet implementation.

Rinse & Repeat!

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Very Tedious, by hand.

$= m(x - x_1) + y_1$

$= f(x_0) + f'(x_0)(x - x_0)$

§3.9 Antiderivatives

Give me ONE position & the derivative & I can build the function!

$f'(x) = x^2 - 3x + 2$ & $f(1) = 7$

what's $f(x)$? Use Antiderivatives!

$$\frac{d}{dx}\left[\frac{x^3}{3}\right] = \frac{1}{3}\frac{d}{dx}\left[x^3\right] = \frac{1}{3}\left[3x^2\right] = x^2 !$$

$$\boxed{f(x) = \frac{1}{3}x^3 - \frac{3}{2}x^2 + 2x + C}$$ for some $C \in \mathbb{R}$

Now, $f(1) = 7$, so...

$$f(1) = \frac{1}{3} - \frac{3}{2} + 2 + C = \frac{2 - 9 + 12}{6} + C = 7$$

$$\frac{5}{6} + C = 7 \longrightarrow$$

$$C = \frac{7}{1}\cdot\frac{6}{6} - \frac{5}{6} = \frac{42 - 5}{6} = \boxed{\frac{37}{6} = C}, \text{ so}$$

$$f(x) = \frac{1}{3}x^3 - \frac{3}{2}x^2 + 2x + \frac{37}{6}$$

$$f(1) = \frac{1}{3} - \frac{3}{2} + 2 + \frac{37}{6} = \frac{1}{3}\cdot\frac{2}{2} - \frac{3}{2}\cdot\frac{3}{3} + \frac{2}{1}\cdot\frac{6}{6} + \frac{37}{6}$$

$$= \frac{2 - 9 + 12 + 37}{6} = \frac{42}{6} = 7 \checkmark$$

$$f'(x) = x^2 - 3x + 2 \checkmark$$

Same Deal for $f''(x)$, with 2 conditions

Boundary Conditions $f(1)$ & $f(7)$ given

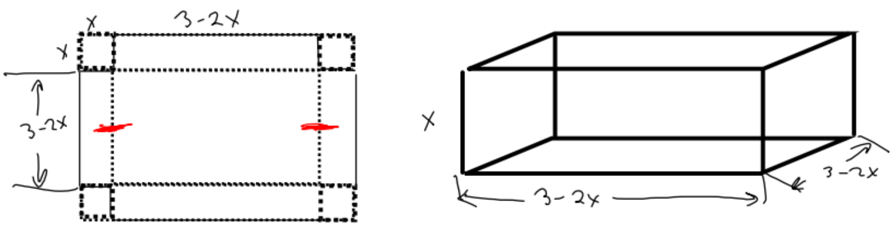
OR

Initial Conditions. $f'(2)$ & $f(2)$ given

OK. That's the overview of 2nd half of Chapter 3.

4. Question Details SCalc8 3.7.012. [3943251]

Consider the following problem: A box with an open top is to be constructed from a square piece of cardboard, 3 ft wide, by cutting out a square from each of the four corners and bending up the sides. Find the largest volume that such a box can have.



$V = x(3-2x)^2 = x(2x-3)^2$

V = Volume (in ft^3) as a function of ...

x = the length of the squares we cut out (in ft)

$V = x(3-2x)^2$ is objective function to be maximized.

$V' = (2x-3)^2 + x(2(2x-3)(2))$

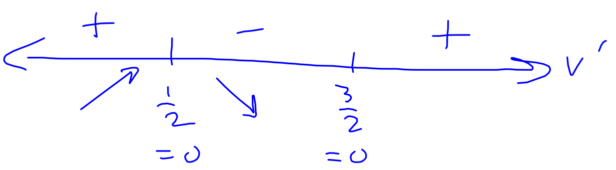
$= (2x-3)(2x-3+4x) = (2x-3)(6x-3) \overset{SET}{=} 0$

$2x-3=0$ OR $6x-3=0$

$x = \frac{3}{2}$ $\quad$ $6x=3$

$\boxed{x = \frac{1}{2}}$

Check:



1st Deriv Test says $V(\frac{1}{2})$ is max!