Today: Examples Related to Take-Home Test.

Graph:

Check your inbox on D2L for the Take-Home Test 3

#1a $f(x) = \sin(x)\cos(x)$

#1b $g(x) = \cos^2(x) - \sin(x)$

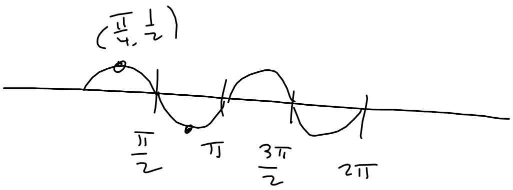
#2 $R(x) = \frac{(x+3)(x-5)}{x-2} = \frac{x^2-2x-15}{x-2}$ Vertical & Oblique Asymptote.

$f' = \cos^2(x) - \sin^2(x)$

$f'' = -2\cos(x)\sin(x) - 2\sin(x)\cos(x) = -4\sin(x)\cos(x)$

$f'' = 2\cos(x)(-\sin(x)) - 2\sin(x)\cos(x) = -4\sin(x)\cos(x)$

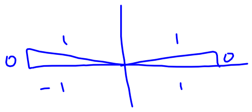
Maple says $f'' \equiv 0$?



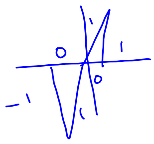
#1a $f(x) = \sin(x)\cos(x) \overset{\text{SET}}{=} 0$

sine & cosine are cnts and differentiable everywhere!

$\Longrightarrow \sin(x) = 0$ OR $\cos(x) = 0$

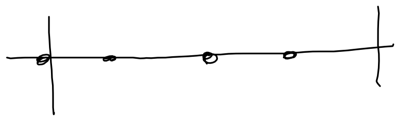


$x = 0, \pi \ (, 2\pi)$



$\frac{\pi}{2}, \frac{3\pi}{2}$

x-ints



$f'(x) = \cos(x)\cos(x) + \sin(x)(-\sin(x)) = \cos^2(x) - \sin^2(x) \overset{\text{SET}}{=} 0$

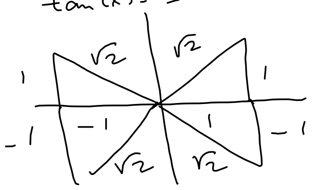
$(fg)' = f' \quad g \quad + \quad f \quad g'$

One method: $\cos^2(x) = \sin^2(x)$

$1 = \frac{\sin^2(x)}{\cos^2(x)} = \tan^2(x) = 1 \Rightarrow$

$\tan(x) = \pm\sqrt{1} = \pm 1$

$\tan(x) = \pm 1$



$x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

Another Method:

$f'(x) = \cos^2(x) - \sin^2(x) = 1 - \sin^2(x) - \sin^2(x) = \boxed{1 - 2\sin^2(x)} \overset{\text{SET}}{=} 0$

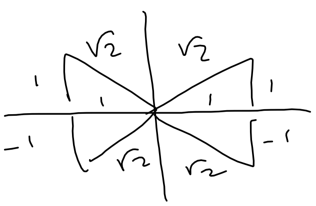
Used this version for f''

$\Rightarrow 2\sin^2(x) - 1 = 0$

$2\sin^2(x) = 1$

$\sin^2(x) = \frac{1}{2}$

$\sin(x) = \pm\frac{1}{\sqrt{2}}$



$x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

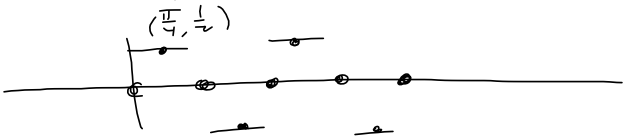
If you're more calculator-dependent, use $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = 45^\circ$ & then convert to π radians via

$(45^\circ)\left(\frac{\pi}{180^\circ}\right) = \frac{\pi}{4}$, etc.

$f\left(\frac{\pi}{4}\right) = \sin\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} = \frac{1}{2} = f\left(\frac{5\pi}{4}\right)$ = Local Max's !

$f\left(\frac{3\pi}{4}\right) = \sin\left(\frac{3\pi}{4}\right)\cos\left(\frac{3\pi}{4}\right) = \frac{1}{\sqrt{2}}\left(-\frac{1}{\sqrt{2}}\right) = -\frac{1}{2}$

$= f\left(\frac{7\pi}{4}\right)$ Local Mins !



$f''(x) = \frac{d}{dx}\left[1 - 2\sin^2(x)\right] = -4\sin(x)\cos(x) \overset{SET}{=} 0 \Rightarrow$

$x = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2} \ (2\pi)$

Lay this info out :

Test $f'\left(\frac{\pi}{6}\right) = 1 - 2\sin^2\left(\frac{\pi}{6}\right)$

$= 1 - 2\left(\frac{1}{2}\right)^2$

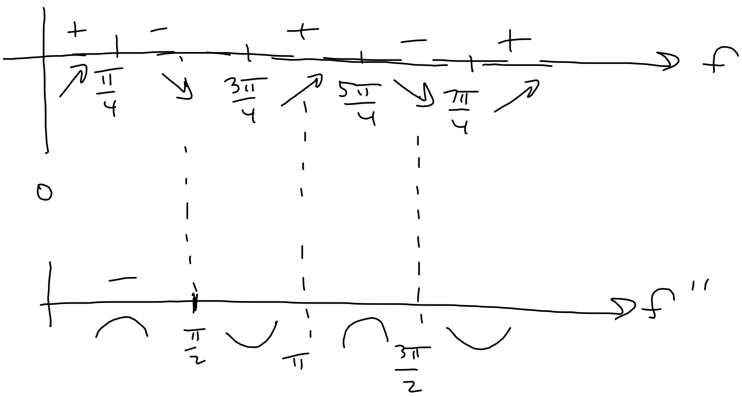
$= 1 - \frac{1}{2} = \frac{1}{2} > 0$

$+ \nearrow$

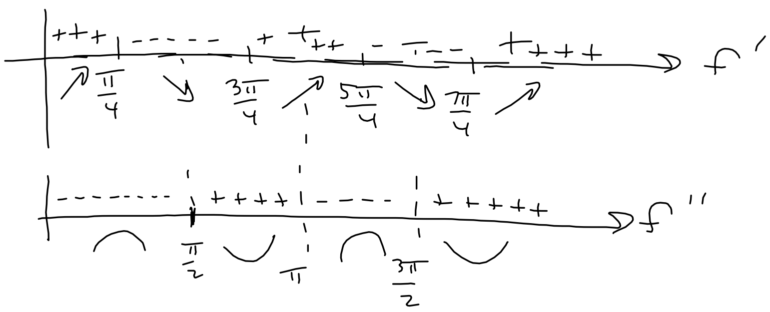
$f'\left(\frac{\pi}{2}\right) = 1 - 2\sin^2\left(\frac{\pi}{2}\right)$

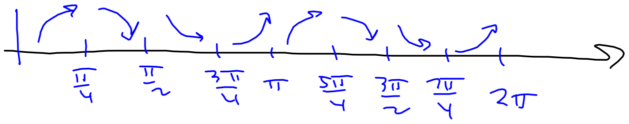
$= -1 < 0$

$- \searrow$

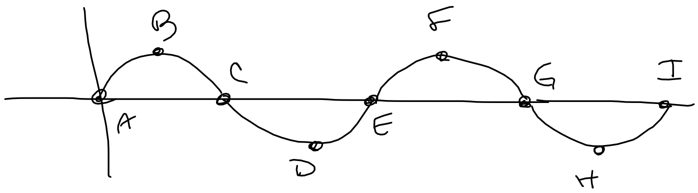


$-4\sin\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{4}\right) = -4(+)(+) < 0 \quad -$





COMBINE !



MAKE A LEGEND

$A = (0,0)$

$B = (\frac{\pi}{4}, \frac{1}{2})$ MAX

$C = (\frac{\pi}{2}, 0)$ I.P.

$D = (\frac{3\pi}{4}, -\frac{1}{2})$ MIN

$E = (\pi, 0)$ IP

$F = (\frac{5\pi}{4}, \frac{1}{2})$

$G = (\frac{3\pi}{2}, 0)$ I.P.

$H = (\frac{7\pi}{4}, -\frac{1}{2})$ MIN

$\left(I = (2\pi, 0) \right)$

#1b $g(x) = \cos^2(x) - \sin(x) = 1 - \sin^2(x) - \sin(x)$

$= -\sin^2(x) - \sin(x) + 1 \overset{\text{SET}}{=} 0 \Longrightarrow$

$\sin^2(x) + \sin(x) - 1 = 0$

Let $u = \sin(x)$: $u^2 + u - 1 = 0$

$u^2 + u + \left(\frac{1}{2}\right)^2 = 1 + \frac{1}{4} = \frac{5}{4}$

$\left(u + \frac{1}{2}\right)^2 = \frac{5}{4} \Longrightarrow$

$u + \frac{1}{2} = \pm\frac{\sqrt{5}}{2} \Longrightarrow$

$u = \frac{-1 \pm \sqrt{5}}{2} = \sin(x)$

$a = 1,\ b = 1,\ c = -1$

$b^2 - 4ac = 1 - 4(1)(-1) = 5$

$x = \frac{-1 \pm \sqrt{5}}{2}$

Note $2 < \sqrt{5} < 3$ Numeracy

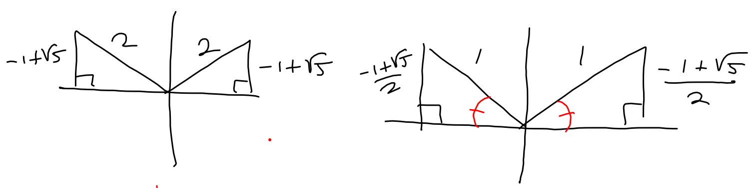
$\frac{-1-\sqrt{5}}{2} = \frac{-3.\text{something}}{2} < -1$

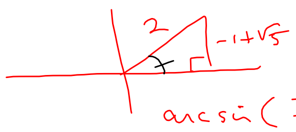
$\frac{-1-\sqrt{5}}{2} \neq \sin(x)$, EVER!

That leaves $\frac{-1+\sqrt{5}}{2} = \sin(x)$

Teacher will want EXACT solutions, before seeing decimal approximations. Here's how it's done:

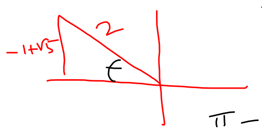
$\sin(x) = \frac{-1+\sqrt{5}}{2}$





$\arcsin\left(\frac{-1+\sqrt{5}}{2}\right)$

$= \sin^{-1}\left(\frac{\sqrt{5}-1}{2}\right)$ DIRECT FROM CALCULATOR.



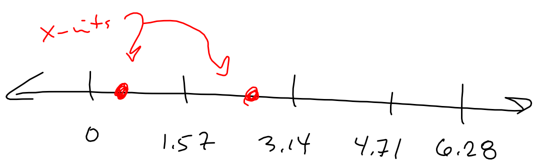
$\pi - \arcsin\left(\frac{\sqrt{5}-1}{2}\right)$!

$\left(\arcsin\left(\frac{\sqrt{5}-1}{2}\right), 0\right)$, $\left(\pi - \arcsin\left(\frac{\sqrt{5}-1}{2}\right), 0\right)$ are the EXACT x-intercepts

(Decimal Approx: (

(.6662394315 , 0) , (2.475353222 , 0)

Decimal radians:



#1b $g(x) = \cos^2(x) - \sin(x)$

$g'(x) = 2\cos(x)(-\sin(x)) - \cos(x)$

$= \cos(x)\left[-2\sin(x) - 1\right] \overset{SET}{=} 0$

$\frac{d}{dx}\left[(\cos(x))^2\right]$

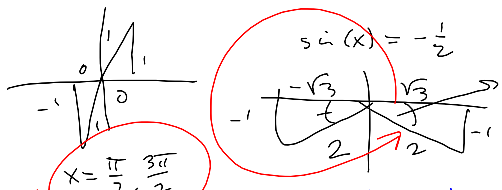
$= 2\cos(x)\cdot\frac{d}{dx}\left[\cos(x)\right]$

Chain Rule!

$\cos(x) = 0$

$2\sin(x) + 1 = 0$

$\sin(x) = -\frac{1}{2}$



$x = \frac{\pi}{2}, \frac{3\pi}{2}$

$30° = \frac{\pi}{6}$

$x = \frac{7\pi}{6}, \frac{11\pi}{6}$

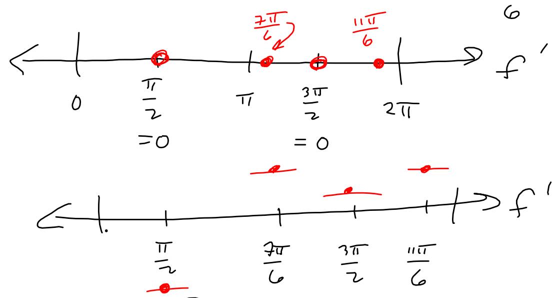
Calculator says "$\frac{\pi}{2}$" or "90°"

Calculator says: $\sin^{-1}(-\frac{1}{2}) = -30°$ OR $-\frac{\pi}{6}$ Radians

You say: $360° - 30° = 330° = \frac{11\pi}{6}$

& $180° + 30° = 210° = \frac{7\pi}{6}$

Looking 4 solutions in $[0, 2\pi)$



$f(\frac{\pi}{2}) = \cos^2(\frac{\pi}{2}) - \sin(\frac{\pi}{2}) = -1$ MIN?

$f(\frac{3\pi}{2}) = \cos^2(\frac{3\pi}{2}) - \sin(\frac{3\pi}{2}) = -(-1) = +1$ (MIN?)

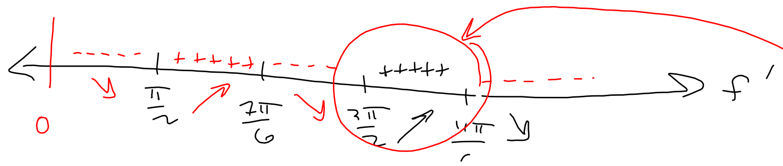
$f(\frac{7\pi}{6}) = \cos^2(\frac{7\pi}{6}) - \sin(\frac{7\pi}{6})$

$= (-\frac{\sqrt{3}}{2})^2 - (-\frac{1}{2}) = \frac{3}{4} + \frac{1}{2}\cdot\frac{2}{2} = \frac{5}{4}$ MAX?

$f(\frac{11\pi}{6}) =$ SAME $= \frac{5}{4}$ (MAX)?

Critical: $x = \frac{\pi}{2}, \frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6}$

Test: $x = \frac{\pi}{4}, \pi, \frac{8\pi}{6} = \frac{4\pi}{3}, 335°$?!



$$\left[fp\left(\frac{\text{Pi}}{4}\right), fp(\text{Pi}), fp\left(\frac{4\cdot\text{Pi}}{3}\right), fp\left(\frac{335\cdot\text{Pi}}{180}\right)\right]$$

$$[-1.707106781, 1., -0.3660254040, -0.1402633438]$$

Didn't test between 270° & 330° (i.e., $\frac{3\pi}{2}$ & $\frac{11\pi}{6}$)

Cacie, Carlos, Marylu, Lydia.

Test $\boxed{fp(\frac{5\pi}{3}) > 0}$ + ↗

$\frac{3\pi}{2} < \frac{5\pi}{3} < \frac{11\pi}{6}$

Next time: Finish this one and do the rational function graph.

3/10/21 Finishing

$g'(x) = 2\cos(x)(-\sin(x)) - \cos(x) = -2\sin(x)\cos(x) - \cos(x)$

$\Rightarrow g''(x) = -2\cos^2(x) + 2\sin^2(x) + \sin(x)$

$= -2[1-\sin^2(x)] + 2\sin^2(x) + \sin(x)$

$= -2 + 2\sin^2(x) + 2\sin^2(x) + \sin(x)$

$= 4\sin^2(x) + \sin(x) - 2 \overset{\text{SET}}{=} 0 \Rightarrow$

$a=4,\ b=1,\ c=-2,\ u=\sin(x)$

$b^2-4ac = 1^2-4(4)(-2) = 33$

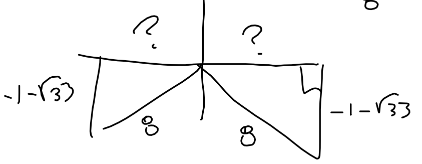
$u = \dfrac{-1\pm\sqrt{33}}{2(4)} = \dfrac{-1\pm\sqrt{33}}{8} = \sin(x)$

$5 < \sqrt{33} < 6$

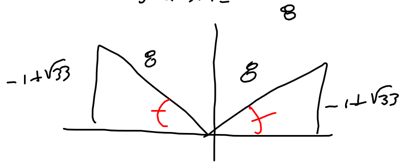
so $\left|\dfrac{-1\pm\sqrt{33}}{8}\right| < 1$

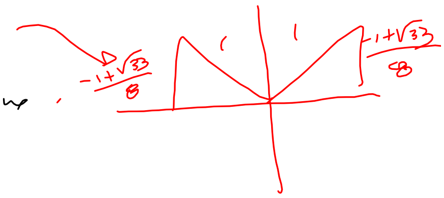
$\Rightarrow$ 4 solns for $\sin(x)$.

$\sin(x) = \dfrac{-1-\sqrt{33}}{8}$



$\sin(x) = \dfrac{-1+\sqrt{33}}{8}$





in $[0,2\pi)$, we have:

$2\pi + \arcsin\left(\frac{-1-\sqrt{33}}{8}\right)$,

$\pi - \arcsin\left(\frac{-1-\sqrt{33}}{8}\right)$

in $[0,2\pi)$, we have:

$\arcsin\left(\frac{-1+\sqrt{33}}{8}\right)$,

$\pi - \arcsin\left(\frac{-1+\sqrt{33}}{8}\right)$

Can we evaluate $g(x)$ @ those values, by hand?

$g(x) = \cos^2(x) - \sin(x)$

$g\left(\pi - \arcsin\left(\frac{-1-\sqrt{33}}{8}\right)\right)$ = exact y-value.

I'll probably just accept this, rather than make you turn the crank.

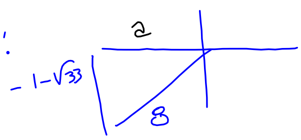
$\left(\cos\left(\pi - \arcsin\left(\frac{-1-\sqrt{33}}{8}\right)\right)\right)^2$

$\cos(u+v) = \cos(u)\cos(v) - \sin(u)\sin(v)$

$= \left(\cos(\pi)\cos\left(\arcsin\left(\frac{-1-\sqrt{33}}{8}\right)\right) - \overset{0}{\sin(\pi)}\sin(\sim)\right)^2$

$= (-1)$

Scratch:



$a^2 = 8^2 - (+1+\sqrt{33})^2$

$= 64 - (1 + 2\sqrt{33} + 33)$

$= 64 - 32 + 2\sqrt{33} = 32 + 2\sqrt{33}$

$\Rightarrow a = \sqrt{32+2\sqrt{33}}$ is EXACT VALUE!

& so $\cos\left(\arcsin\left(\frac{-1-\sqrt{33}}{8}\right)\right) = \dfrac{\sqrt{32+2\sqrt{33}}}{8}$

$$= \left((-1)\left(\frac{\sqrt{32+2\sqrt{33}}}{8}\right)\right)^2 = \frac{32+2\sqrt{33}}{64}$$ is exact value of the $\cos^2(x)$ term!

$$g(\text{mess}) = \cos^2(\text{mess}) - \sin(\text{mess})$$

$$= \frac{32+2\sqrt{33}}{64} - \sin\left(\arcsin\left(\frac{-1-\sqrt{33}}{8}\right)\right)$$

$$= \frac{32+2\sqrt{33}}{64} - \left(\frac{-1-\sqrt{33}}{8}\right) !$$

$$= \frac{32+2\sqrt{33}}{64} + \frac{1+\sqrt{33}}{8}$$

$$= \frac{16+\sqrt{33}}{32} + \frac{4+4\sqrt{33}}{32} = \frac{20+5\sqrt{33}}{32} = g(\text{mess})!$$

I think I'll make exact y-values BONUS.
But I'll want to see the exact x-values before you report the decimal x- & y-values

3/10 CACIE, CARLOS, Alexander.
HARD CORE!

Rational Function from the past

$$R(x) = \frac{6x^2+5x-6}{2x-8} =$$

$\mathcal{D} = \mathbb{R} \setminus \{4\}$ zeros of denom

x-ints: $6x^2+5x-6$

$= 6x^2+9x-4x-6$

$= 3x(2x+3)-2(2x+3)$

$= (2x+3)(3x-2) \overset{\text{SET}}{=} 0$

$\Rightarrow x = -\frac{3}{2}, \frac{2}{3}$

-36

$(9)(-4) = -36$

$9-4=+5$

Let BIGGER mean "bigger degree"

Proper: $\frac{\text{smaller}}{\text{bigger}}$ $y=0$ is H.A.

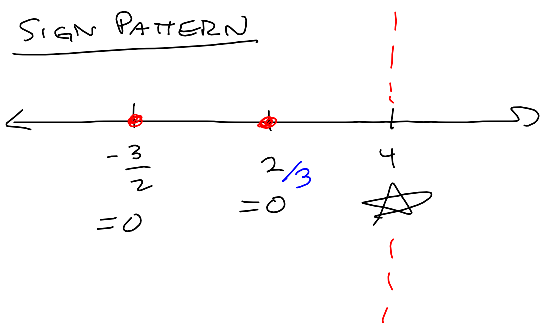
Improper:

$\frac{\text{SAME}}{\text{SAME}}$: $y = \text{const.}$ Look @ leading coefficients.

$\frac{\text{Bigger}}{\text{Smaller}}$: oblique or Slant Asymptotes

Find by long (or synthetic) division.

SIGN PATTERN



Proper

$\frac{3x^2+2x}{x^3-1}$: H.A. $\boxed{y=0}$

Improper

$\frac{3x^2+2x}{5x^2-7}$: H.A. $\boxed{y=\frac{3}{5}}$

We've got:

$\frac{6x^2+5x-6}{2x-8}$ SLANT!

Slant Asymptote:

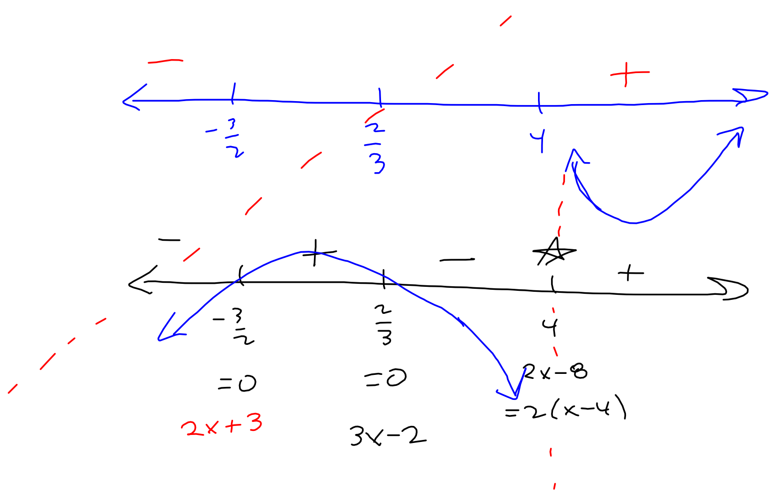
Long Division:

$$\begin{array}{r} 3x + \frac{29}{2} + r \ldots \\ 2x-8 \overline{) \; 6x^2+5x-6} \\ -(6x^2-24x) \\ \hline 29x-6 \end{array}$$

$\frac{6x^2}{2x} = 3x$

$\frac{29x}{2x} = \frac{29}{2}$

$y = 3x + \frac{29}{2}$ is slant asymptote



Just from what Slant As. tells us!

All from college algebra!

$$R(x) = \frac{6x^2+5x-6}{2x-8}$$

$$\begin{array}{r} 3x + \frac{29}{2} \quad r\ 110 \\ 2x-8 \overline{\left) 6x^2+5x-6 \right.} \\ -(6x^2-24x) \quad\quad \\ \hline 29x-6 \\ -(29x-116) \\ \hline 110 \end{array}$$

This says:

$$R(x) = 3x + \frac{29}{2} + \frac{110}{2x-8} \ !$$

$$= 3x + \frac{29}{2} + \frac{55}{x-4} \ !$$

This might be easier to differentiate!

$$\Rightarrow R'(x) = 3 - \frac{55}{(x-4)^2}$$

$$\frac{d}{dx}\left[55(x-4)^{-1}\right] = -55(x-4)^{-2}$$

$$\& \ R''(x) = \frac{2 \cdot 55}{(x-4)^3} = \frac{110}{(x-4)^3}$$

$$\Rightarrow R'(x) = 3 - \frac{55}{(x-4)^2} \quad \& \ R''(x) = \frac{2 \cdot 55}{(x-4)^3} = \frac{110}{(x-4)^3}$$

$R'(x) = 0$

$$\frac{55}{(x-4)^2} = 3$$

$$3(x-4)^2 = 55$$

$$(x-4)^2 = \frac{55}{3}$$

$$x - 4 = \pm\sqrt{\frac{55}{3}}$$

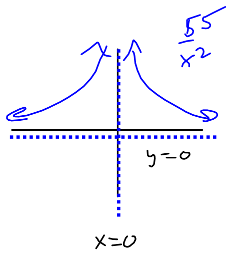
$$x = 4 \pm \sqrt{\frac{55}{3}}$$

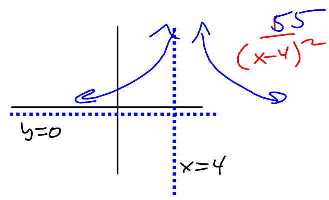
Also, $R'(x) \not\exists$ @ $x = 4$

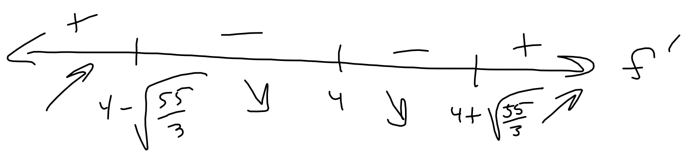
(Don't forget this!)

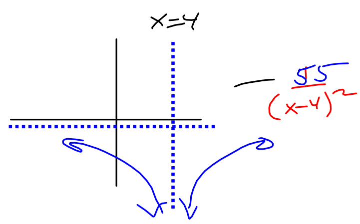
$R''(x) = 0$ Never!

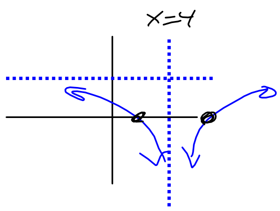
But $f''(x) \not\exists$ @ $x = 4$











$R'(x)$ the other way :

$$R(x) = \frac{6x^2+5x-6}{2x-8} = \frac{f}{g} \Rightarrow$$

$$R'(x) = \left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2} = \frac{(12x+5)(2x-8) - (6x^2+5x-6)(2)}{(2x-8)^2}$$

$$= \frac{24x^2 - 96x + 10x - 40 - 12x^2 - 10x + 12}{(2x-8)^2}$$

$$= \frac{12x^2 - 122x - 28}{(2x-8)^2}$$

$$\frac{2(6x^2 - 61x - 14)}{4(x-4)^2}$$

$$= \frac{6x^2 - 61x - 14}{2(x-4)^2}$$

$$= (2(x-4))^2$$
$$= 2^2(x-4)^2$$

$$\frac{6x^2 - 61x - 14}{2(x^2 - 8x + 16)}$$? Don't Know!

I'll write this up & distribute it.

Due After Spring Break.