To day: Examples Related to Take-Home Test.

Graph:

Check your inbox on D2L for the Take-Home Test 3

#1a $f(x) = \sin(x)\cos(x)$

#1b $g(x) = \cos^2(x) - \sin(x)$

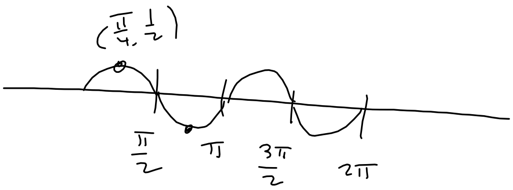
#2 $R(x) = \dfrac{(x+3)(x-5)}{x-2} = \dfrac{x^2-2x-15}{x-2}$ Vertical & Oblique Asymptote.

$f' = \cos^2(x) - \sin^2(x)$

$f'' = -2\cos(x)\sin(x) - 2\sin(x)\cos(x) = -4\sin(x)\cos(x)$

$f'' = 2\cos(x)(-\sin(x)) - 2\sin(x)\cos(x) = -4\sin(x)\cos(x)$

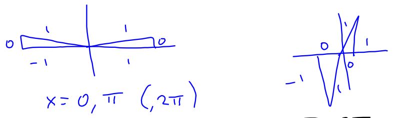
Maple says $f'' \equiv 0$?



#1a $f(x) = \sin(x)\cos(x) \overset{SET}{=} 0$

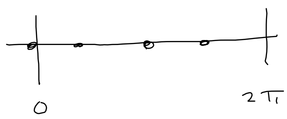
sine & cosine are cnts and differentiable everywhere!

$\Rightarrow \sin(x) = 0$ OR $\cos(x) = 0$



$x = 0, \pi \ (, 2\pi)$ $\quad \frac{\pi}{2}, \frac{3\pi}{2}$

x-ints



$f'(x) = \cos(x)\cos(x) + \sin(x)(-\sin(x)) = \cos^2(x) - \sin^2(x) \overset{SET}{=} 0$

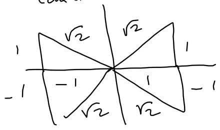
$(fg)' = f' \ g + f \ g'$

One method: $\cos^2(x) = \sin^2(x)$

$1 = \frac{\sin^2(x)}{\cos^2(x)} = \tan^2(x) = 1 \Rightarrow$

$\tan(x) = \pm\sqrt{1} = \pm 1$

$\tan(x) = \pm 1$



$x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

Another Method:

$f'(x) = \cos^2(x) - \sin^2(x) = 1 - \sin^2(x) - \sin^2(x) = \boxed{1 - 2\sin^2(x)} \overset{SET}{=} 0$

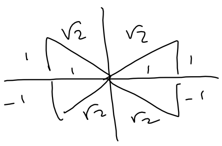
used this version for f''

$\Rightarrow 2\sin^2(x) - 1 = 0$

$2\sin^2(x) = 1$

$\sin^2(x) = \frac{1}{2}$

$\sin(x) = \pm \frac{1}{\sqrt{2}}$



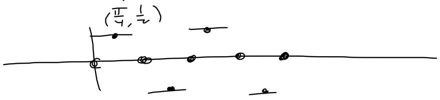
$x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

If you're more calculator-dependent, use $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = 45°$ & then convert to π radians via $(45°)\left(\frac{\pi}{180°}\right) = \frac{\pi}{4}$, etc.

$f\left(\frac{\pi}{4}\right) = \sin\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} = \frac{1}{2} = f\left(\frac{5\pi}{4}\right)$ = Local Max's!

$f\left(\frac{3\pi}{4}\right) = \sin\left(\frac{3\pi}{4}\right)\cos\left(\frac{3\pi}{4}\right) = \frac{1}{\sqrt{2}}\left(-\frac{1}{\sqrt{2}}\right) = -\frac{1}{2}$

$= f\left(\frac{7\pi}{4}\right)$ Local Mins!



$f''(x) = \frac{d}{dx}\left[1 - 2\sin^2(x)\right] = -4\sin(x)\cos(x) \overset{SET}{=} 0 \Rightarrow$

$x = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2} \ (, 2\pi)$

lay this info out:

Test $f'\left(\frac{\pi}{6}\right) = 1 - 2\sin^2\left(\frac{\pi}{6}\right)$

$= 1 - 2\left(\frac{1}{2}\right)^2$

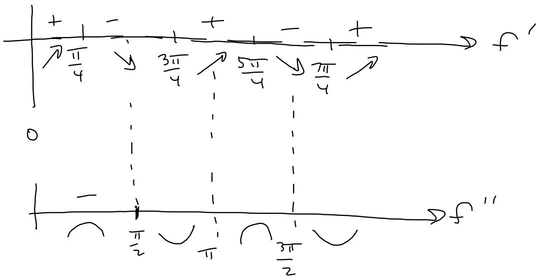
$= 1 - \frac{1}{2} = \frac{1}{2} > 0$

$+ \nearrow$

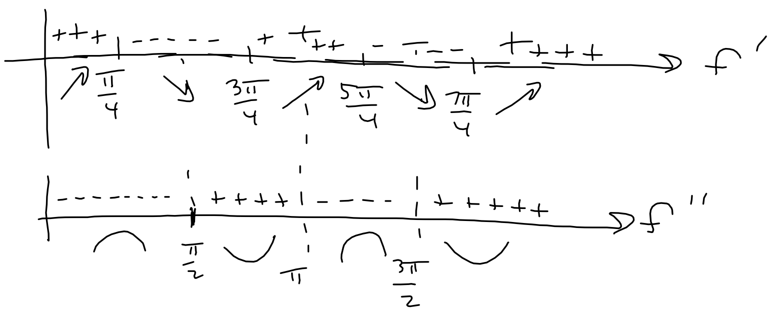
$f'\left(\frac{\pi}{2}\right) = 1 - 2\sin^2\left(\frac{\pi}{2}\right)$

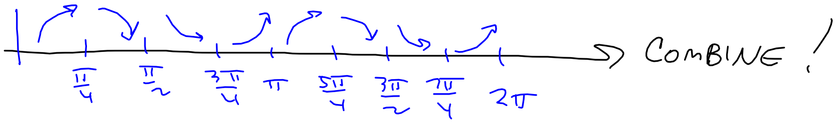
$= -1 < 0$

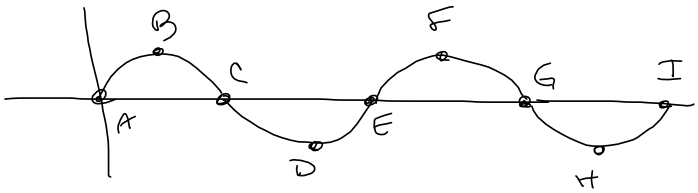
$- \searrow$



$-4\sin\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{4}\right) = -4(+)(+) < 0 \quad -$







MAKE A LEGEND

$A = (0,0)$

$B = (\frac{\pi}{4}, \frac{1}{2})$ MAX

$C = (\frac{\pi}{2}, 0)$ I.P.

$D = (\frac{3\pi}{4}, -\frac{1}{2})$ MIN

$E = (\pi, 0)$ I P

$F = (\frac{5\pi}{4}, \frac{1}{2})$

$G = (\frac{3\pi}{2}, 0)$ I.P.

$H = (\frac{7\pi}{4}, -\frac{1}{2})$ MIN

$(I = (2\pi, 0))$

#1b $g(x) = \cos^2(x) - \sin(x) = 1 - \sin^2(x) - \sin(x)$

$= -\sin^2(x) - \sin(x) + 1 \overset{\text{SET}}{=} 0 \implies$

$\sin^2(x) + \sin(x) - 1 = 0$

Let $u = \sin(x)$: $u^2 + u - 1 = 0$

$u^2 + u + \left(\frac{1}{2}\right)^2 = 1 + \frac{1}{4} = \frac{5}{4}$

$\left(u + \frac{1}{2}\right)^2 = \frac{5}{4} \implies$

$u + \frac{1}{2} = \pm\frac{\sqrt{5}}{2} \implies$

$u = \frac{-1 \pm \sqrt{5}}{2} = \sin(x)$

$a = 1,\ b = 1,\ c = -1$

$b^2 - 4ac = 1 - 4(1)(-1) = 5$

$x = \frac{-1 \pm \sqrt{5}}{2}$

Note $2 < \sqrt{5} < 3$

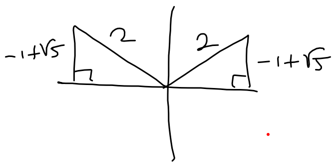
$\frac{-1-\sqrt{5}}{2} = \frac{-2.\text{something}}{2} < -1$

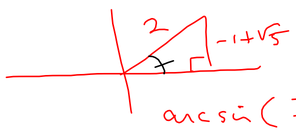
$\frac{-1-\sqrt{5}}{2} \neq \sin(x)$, EVER!

That leaves $\frac{-1+\sqrt{5}}{2} = \sin(x)$

Teacher will want EXACT solutions, before seeing decimal approximations. Here's how it's done:

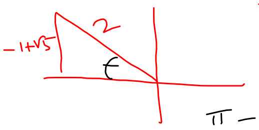
$\sin(x) = \frac{-1+\sqrt{5}}{2}$





$\arcsin\left(\frac{-1+\sqrt{5}}{2}\right)$

$= \sin^{-1}\left(\frac{\sqrt{5}-1}{2}\right)$ DIRECT FROM CALCULATOR.



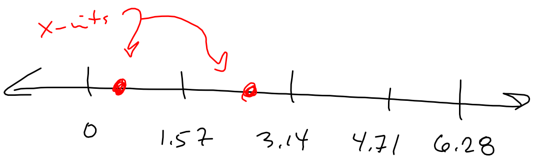
$\pi - \arcsin\left(\frac{\sqrt{5}-1}{2}\right)$!

$\left(\arcsin\left(\frac{\sqrt{5}-1}{2}\right), 0\right)$, $\left(\pi - \arcsin\left(\frac{\sqrt{5}-1}{2}\right), 0\right)$ are the EXACT x-intercepts

(Decimal Approx: (

$(.6662394315, 0)$, $(2.475353222, 0)$)

Decimal radians:



#1b $g(x) = \cos^2(x) - \sin(x)$

$g'(x) = 2\cos(x)(-\sin(x)) - \cos(x)$

$= \cos(x)\left[-2\sin(x) - 1\right] \overset{SET}{=} 0$

$\frac{d}{dx}\left[(\cos(x))^2\right]$

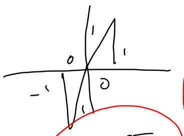
$= 2\cos(x)\cdot\frac{d}{dx}\left[\cos(x)\right]$

Chain Rule!

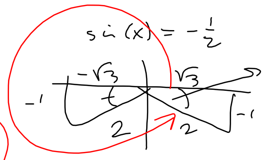
$\cos(x) = 0$

$2\sin(x) + 1 = 0$

$\sin(x) = -\frac{1}{2}$



$x = \frac{\pi}{2}, \frac{3\pi}{2}$



$30^\circ = \frac{\pi}{6}$

$x = \frac{7\pi}{6}, \frac{11\pi}{6}$

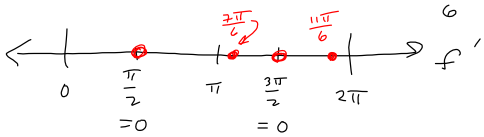
Calculator says: $\sin^{-1}\left(-\frac{1}{2}\right) = -30^\circ$

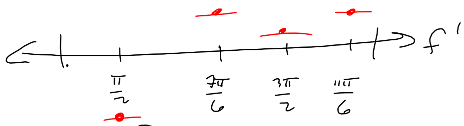
OR $-\frac{\pi}{6}$ Radians

Calculator says "$\frac{\pi}{2}$" or "90°"

You say: $360^\circ - 30^\circ = 330^\circ = \frac{11\pi}{6}$

& $180^\circ + 30^\circ = 210^\circ = \frac{7\pi}{6}$





$f\left(\frac{\pi}{2}\right) = \cos^2\left(\frac{\pi}{2}\right) - \sin\left(\frac{\pi}{2}\right) = -1$ MIN ?

$f\left(\frac{3\pi}{2}\right) = \cos^2\left(\frac{3\pi}{2}\right) - \sin\left(\frac{3\pi}{2}\right) = -(-1) = +1$ (MIN?)

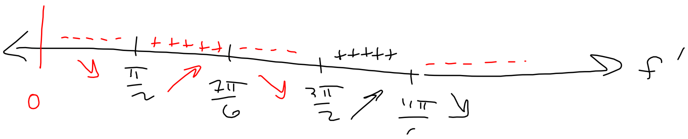
$f\left(\frac{7\pi}{6}\right) = \cos^2\left(\frac{7\pi}{6}\right) - \sin\left(\frac{7\pi}{6}\right)$

$= \left(-\frac{\sqrt{3}}{2}\right)^2 - \left(-\frac{1}{2}\right) = \frac{3}{4} + \frac{1}{2}\cdot\frac{2}{2} = \frac{5}{4}$ MAX ?

$f\left(\frac{11\pi}{6}\right) =$ SAME $= \frac{5}{4}$ (MAX) ?

Critical: $x = \frac{\pi}{2}, \frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6}$

Test: $x = \frac{\pi}{4}, \pi, \frac{8\pi}{6} = \frac{4\pi}{3}, 335^\circ$?!



$$\left[fp\left(\frac{\text{Pi}}{4}\right), fp(\text{Pi}), fp\left(\frac{4\cdot\text{Pi}}{3}\right), fp\left(\frac{335\cdot\text{Pi}}{180}\right)\right]$$

[−1.707106781, 1., −0.3660254040, −0.1402633438]

Didn't test between 270° & 330° (i.e., $\frac{3\pi}{2}$ & $\frac{11\pi}{6}$)

Cacie, Carlos, Marylu, Lydia.

Test $fp\left(\frac{5\pi}{3}\right) > 0$ +

Next time: Finish this one and do the rational function graph.