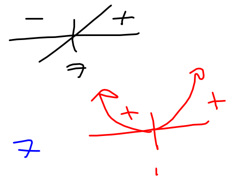
Questions?   §3.3 $f(x) \xrightarrow{x \to \infty} x^2$ sort of

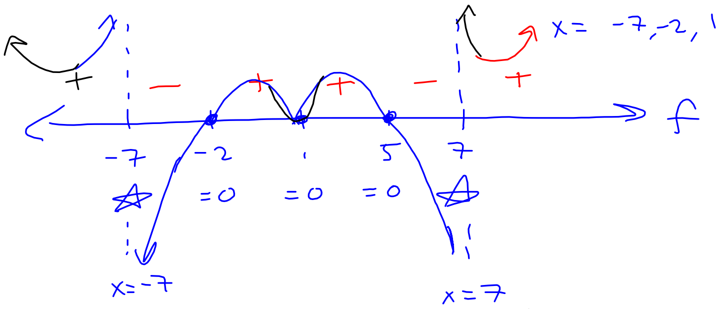
Quick Review:

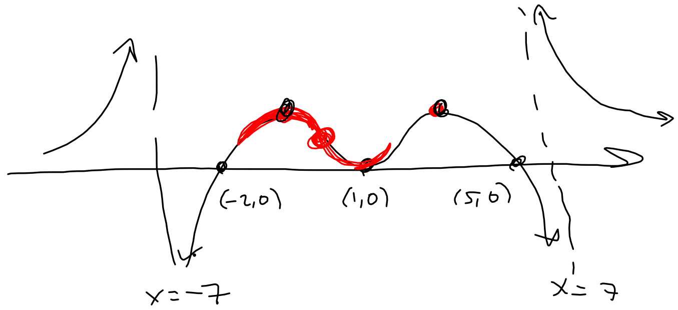
Quick sketch of $f(x) = \dfrac{(x-1)^2(x+2)^3(x-5)}{(x+7)^3(x-7)}$



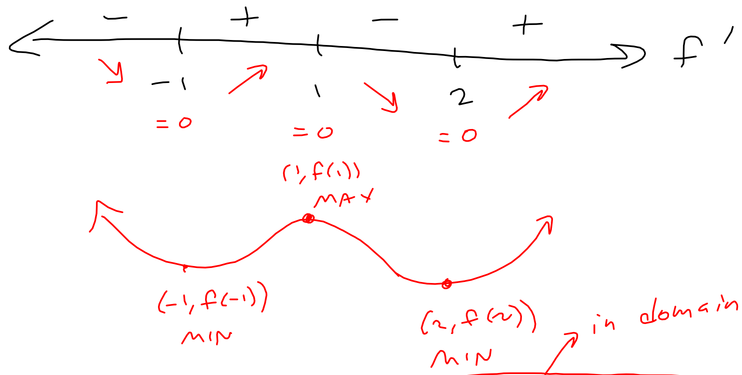
using sign pattern: Important: $x = 1, -2, 5, -7, 7$

$x = -7, -2, 1, 5, 7$





Suppose I said this was the sign pattern for f':



in domain of f

Book's going to focus on critical values if you want to know those, for sure, but interesting things also happen at asymptotes. → not in domain of f, but still be important for building sign pattern of f' & f''

$f(x) = \frac{1}{(x-3)^2} = (x-3)^{-2}$

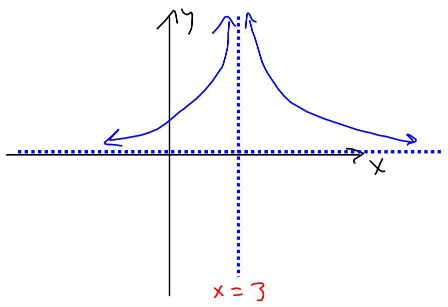
$x=3$ V.A.

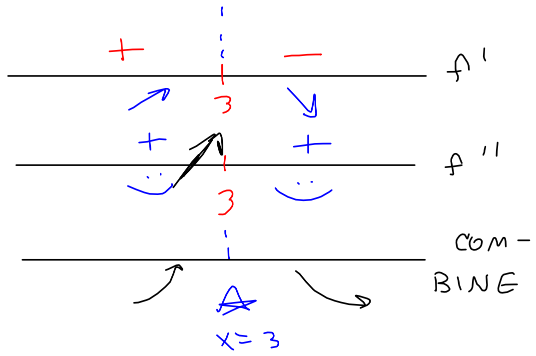
$f'(x) = -2(x-3)^{-3} = \frac{-2}{(x-3)^3}$

f' $\nexists$ @ $x=3$

$f''(x) = 6(x-3)^{-4} = \frac{6}{(x-3)^4}$

f'' $\nexists$ @ $x=3$





f'' is all about attitude!

+: $f'' > 0$

− $f'' < 0$

§3.3 1^{st} & 2^{nd} Derivatives' effects on a graph.

E7 $f(x) = x^{2/3}(6-x)^{1/3}$, $f'(x) = \dfrac{(4-x)'}{x^{1/3}(6-x)^{2/3}}$, $f''(x) = \dfrac{-8}{x^{4/3}(6-x)^{5/3}}$

$x^{4/3}$ Does NOT CHANGE SIGN!

@ $x=7$ $+ = \dfrac{-}{(+)(-)}$

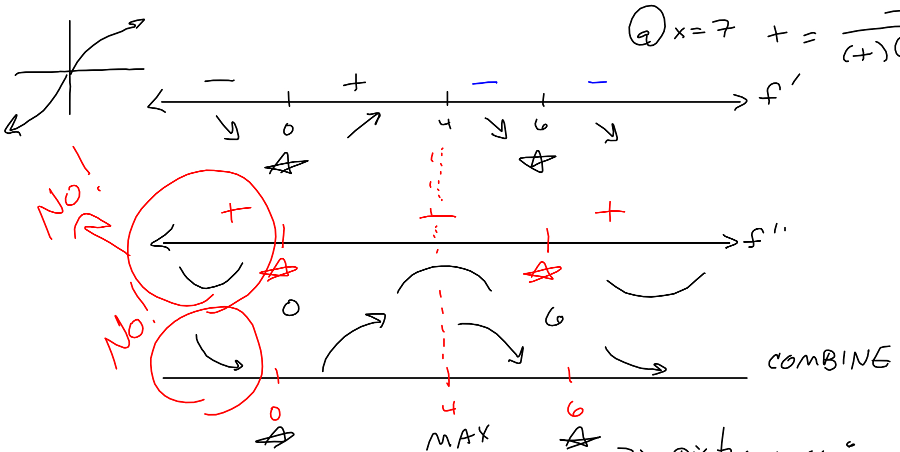
$x^{2/3}$ shape

$(6-x)^{2/3}$

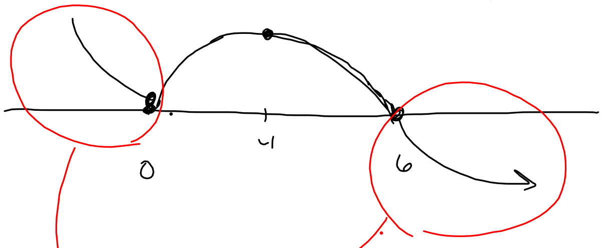
Test Value:

$f'(7) = \dfrac{4-7}{7^{1/3}\left((6-7)^2\right)^{1/3}}$

< 0



Big-picture way:



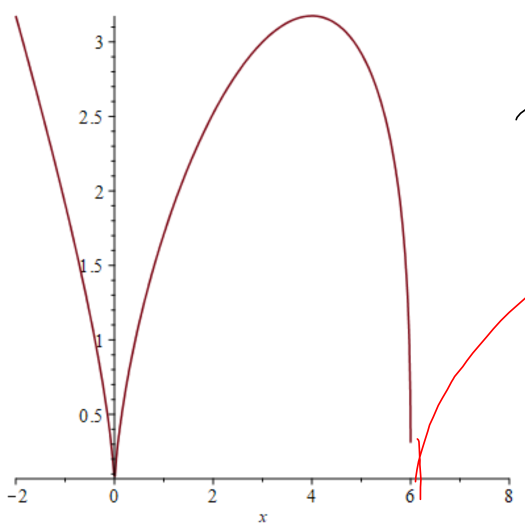
CAS doesn't like $(-)^{1/3}$

$f'=0$: $x=4$ Sign pattern:

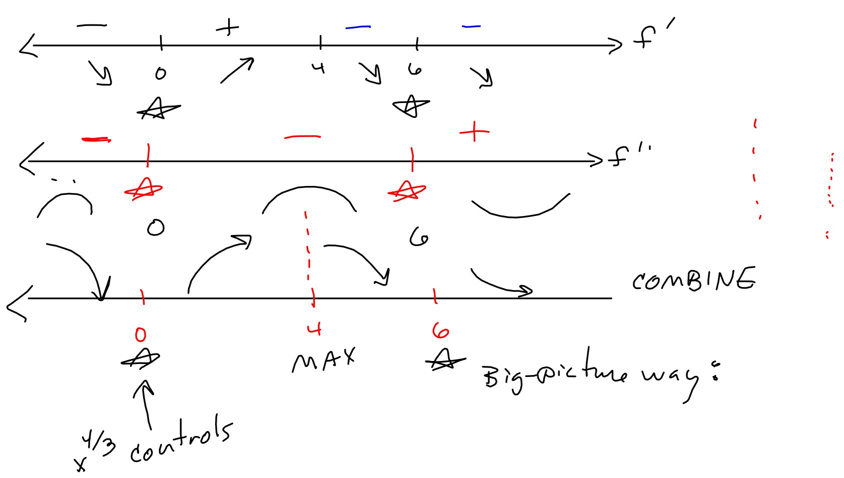
f' ✩: $x=0, 6$

$f'<0$ for BIG x

$\dfrac{4-\text{BIG}}{\text{Big}^{2/3}(6-\text{Big})^{2/3}}$

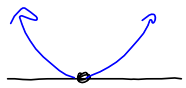


Computer-Algebra Graph helped me find my error to the left of $x=0$.



§ 3.3 2nd Derivative Test

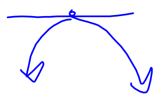
$f'(c) = 0$ & $f''(c) > 0$



MIN!

$f'(c) = 0$ & $f''(c) < 0$

MAX!

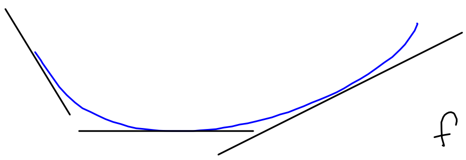


Look for zeros & blowups!

numerator = 0 Denominators = 0

for f' & f''

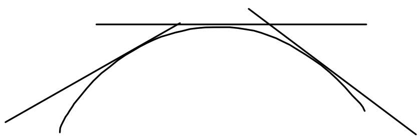
Concave up $f'' > 0$



f lives above its tangents

(Tangent-Line Approximation is an under-estimate!)

Concave Down $f'' < 0$



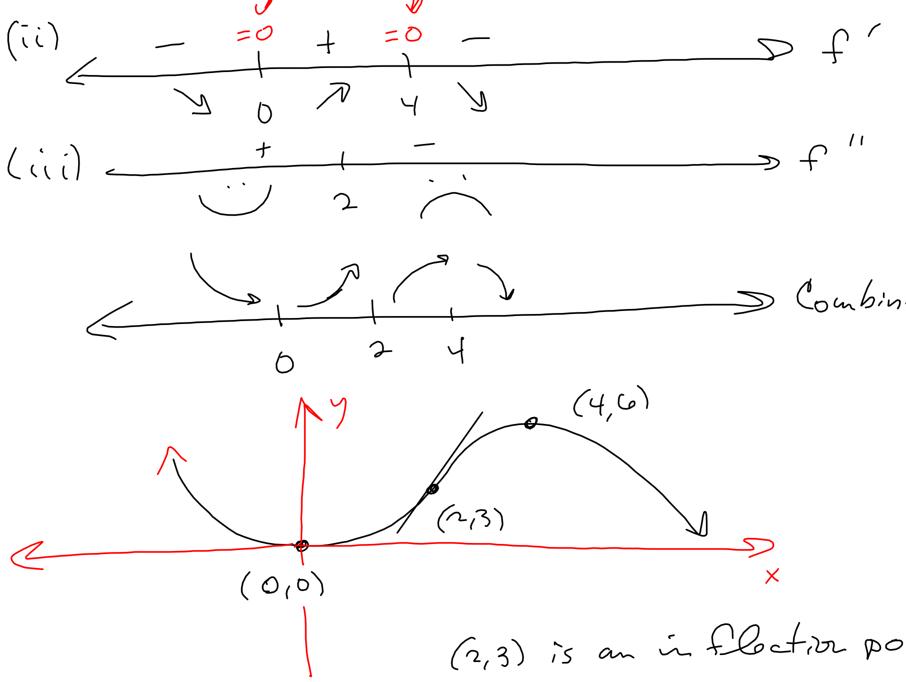
Tangent-line <u>over</u>-estimates $f(x)$.

EXAMPLE 5 Sketch a possible graph of a function f that satisfies the following conditions:

(i) $f(0) = 0, \quad f(2) = 3, \quad f(4) = 6, \quad f'(0) = f'(4) = 0$

(ii) $f'(x) > 0$ for $0 < x < 4, \quad f'(x) < 0$ for $x < 0$ and for $x > 4$

(iii) $f''(x) > 0$ for $x < 2, \quad f''(x) < 0$ for $x > 2$



(2,3) is an inflection point, where concavity changes from up to down. Note: Tangent line crosses f(x) @ inflection point.