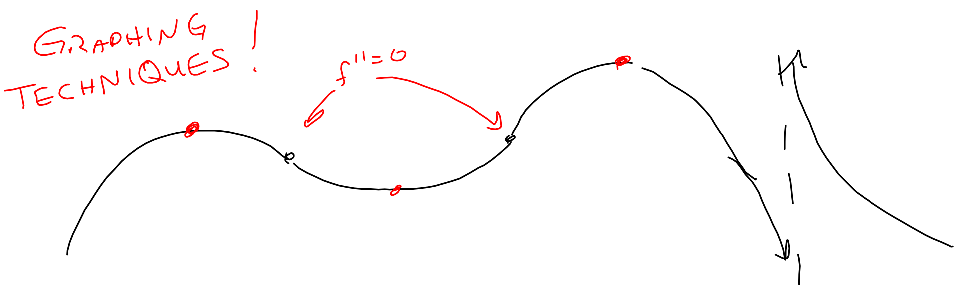
From now on:

There'll be a WebAssign Practice Test and a *very similar* regular test.

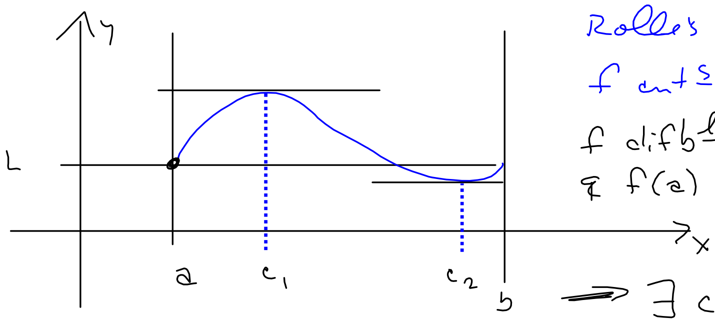
For Take-Home portion of Test 3, use old test 3's from harryzaims.com



No 3.1 Questions?

§3.2 Mean Value Theorem.

Rolle's Theorem (Lemma)



Rolle's:

f cnt^s on $[a,b]$

f difb^l on (a,b)

& $f(a) = f(b) = L$

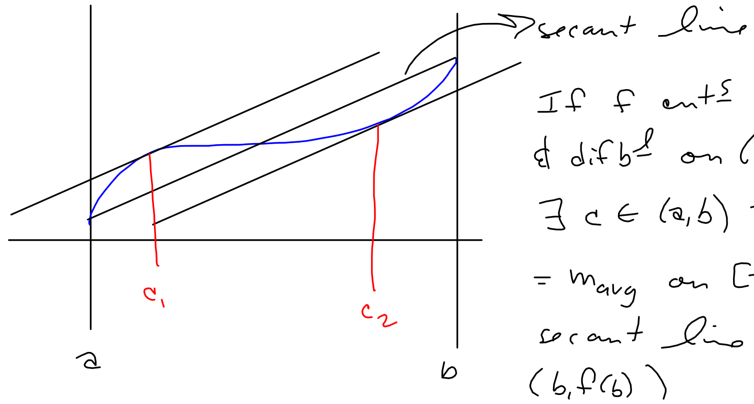
$\Rightarrow \exists\, c \in (a,b) \ni f'(c) = 0$

Proof:

EVT says f achieves its abs max & min values on $[a,b]$. If $f(x) =$ constant, then $f(a) = f(b) =$ max $=$ min

$\Rightarrow f$ is horizontal line with slope $m = 0 = f'(x)$, so $c =$ any # in (a,b).

If f isn't constant, then it has a local min or max somewhere in (a,b). By Fermat's Thm, $f'(c) = 0$ at that max/min.



MEAN VALUE THEOREM

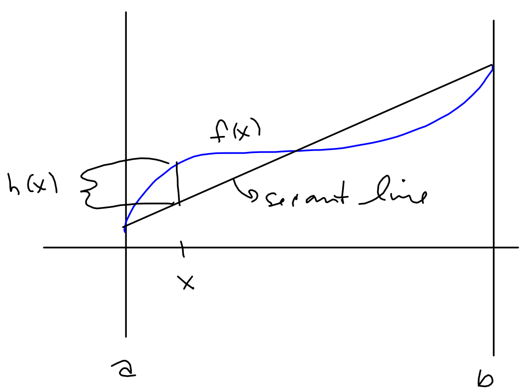
If f cnt^s on $[a,b]$ & difb^l on (a,b) then

$\exists\, c \in (a,b) \ni f'(c) = \dfrac{f(b)-f(a)}{b-a}$

$= m_{avg}$ on $[a,b]$ = slope of secant line from $(a, f(a))$ to $(b, f(b))$

Proof We define

$h(x)$ = Difference between $f(x)$ & the secant line



Secant Line : $(x_1, y_1) = (a, f(a))$

$(x_2, y_2) = (b, f(b)) \Rightarrow$

$m = \dfrac{f(b)-f(a)}{b-a}$ &

$y = m(x - x_1) + y_1$

$y = \dfrac{f(b)-f(a)}{b-a}(x-a) + f(a)$

$$h(x) = f(x) - \frac{f(b)-f(a)}{b-a}(x-a) - f(a)$$

Notice $h(a) = h(b) = 0$

" h is differentiable & continuous everywhere

f is cnt^s on $[a,b]$ & difbl on $(a,b) \Rightarrow$

Rolle's applies.

$\Rightarrow \exists\, c \in (a,b) \ni h'(c) = 0$, i.e.,

Now, $h'(x) = f'(x) - \dfrac{f(b)-f(a)}{b-a}$

$\Rightarrow h'(c) = f'(c) - \dfrac{f(b)-f(a)}{b-a} = 0$

$\Rightarrow f'(c) = \dfrac{f(b)-f(a)}{b-a}$ ■

Other versions :

$f(b) - f(a) = f'(c)(b-a)$ for some $c \in (a,b)$

is useful, later.

$\dfrac{d}{dx}\left(7(x-a)\right)$

$= 7\dfrac{d}{dx}[x-a] = 7$

$f(x) = x^3 - 3x + 2$ on $[-2, 2]$

$$\frac{f(b) - f(a)}{b - a} = \frac{2^3 - 3(2) + 2 - [(-2)^3 - 3(-2) + 2]}{2 - (-2)} = \frac{4 - 0}{4} = 1 = m_{avg}$$

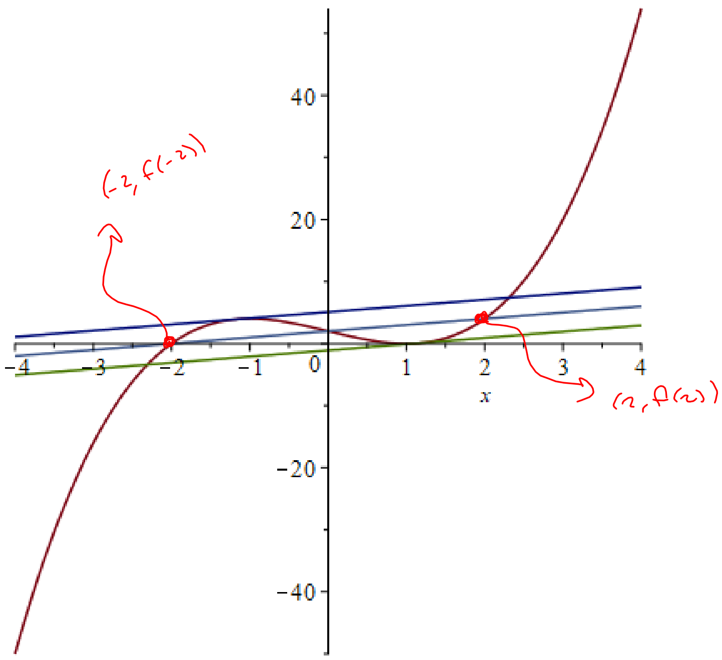
$$f'(x) = 3x^2 - 3 \overset{SET}{=} 1 \longrightarrow$$

$$3x^2 = 4$$

$$\Longrightarrow \quad x^2 = \frac{4}{3}$$

$$\Longrightarrow \quad x = \pm \frac{2}{\sqrt{3}} \text{ OR } \pm \frac{2\sqrt{3}}{3}$$

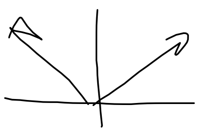
$$c = \pm \frac{2\sqrt{3}}{3}$$



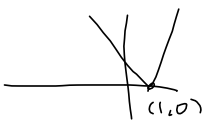
$f(x) = 2 - |2x-1|$ on $[0,3]$

Why no $c \ni f'(c) = \dfrac{f(3)-f(0)}{3-0}$

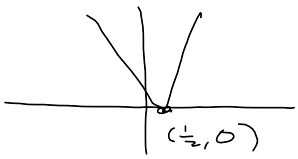
$f(x)$ is not difbl @ $x = \frac{1}{2}$



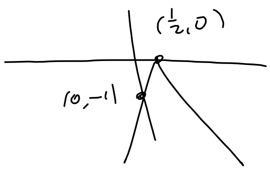
$|x|$



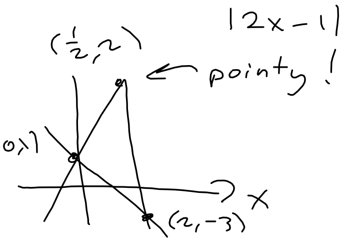
$|x-1|$



$|2x-1|$



$-|2x-1|$



$$f'(x) = \begin{cases} 2 & \text{if } x < \frac{1}{2} \\ -2 & \text{if } x > \frac{1}{2} \end{cases}$$

Show that $2x + \cos(x)$ has exactly one root (zero)

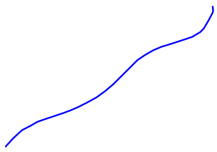
$2(-1) + \cos(-1) < 0$

$2(1) + \cos(1) > 0$

So IVT says $f(x) = 0$ somewhere between -1 & 1

How do we <u>KNOW</u> there can't be MORE than one?

$f'(x) = 2 - \sin(x) > 0 \quad \forall x \in \mathbb{R}$

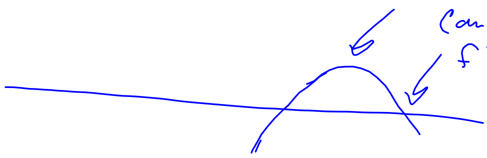


The only way to have MORE ROOTS is for f to have local max somewhere

$f' = 0$

can't happen!

$f' < 0$



$f'(x) = g'(x) \Rightarrow f(x) = g(x) + C$ for some C.