Test 2 – 2 parts :

Test 2 Part I 2.1–2.5 Available Now until Midnight Saturday Night.

Test 2 Part II 2.6–2.9

Tricky making it available 'til Midnight Sunday. I can definitely do it 'til Midnight Saturday. May go in and extend it, if people ask.

2.1 & 2.2 "Derivative by the limit definition" is impossible to administer on line.

Need to know $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = f'(x)$

$\lim_{x\to c} \frac{f(x)-f(c)}{x-c} = f'(c)$

But I won't have you turn the crank on those, in general.

2.2 Tell me what $f, f' \,\&\, f''$ are when you see all 3 graphed together.

2.3

55–58 Find equations of the tangent line and normal line to the curve at the given point.

Perpendicular to tangent line. Intersects " " @ (1,2).

57. $y = \dfrac{3x+1}{x^2+1}$, $(1, 2)$

$$\left(\frac{f}{g}\right)'\Bigg|_{x=1} = \frac{f'g-fg'}{g^2}\Bigg|_{x=1}$$

$$= \frac{3(x^2+1)-(3x+1)(2x)}{(x^2+1)^2}\Bigg|_{x=1} = \frac{3(1^2+1)-(3(1)+1)(2(1))}{(1^2+1)^2}$$

$$= \frac{3(2)-4(2)}{2^2} = \frac{6-8}{4} = \frac{-2}{4} = -\frac{1}{2} = m. \quad (x_1,y_1)=(1,2)$$

$y = m(x-x_1)+y_1$

Tangent Line

$$y = -\tfrac{1}{2}(x-1)+2 = -\tfrac{1}{2}x+\tfrac{1}{2}+2 = \boxed{-\tfrac{1}{2}x+\tfrac{5}{2}}$$

WebAssign wants this

Normal Line: $m_\perp = -\dfrac{1}{m}$

$m = -\frac{1}{2} \Rightarrow$

$m_\perp = +2 \Rightarrow y = 2(x-1)+2$

$= 2x-2+2 = \boxed{2x}$ ⇒ what WebAssign wants

59–62 Find the first and second derivatives of the function.

61. $f(x) = \dfrac{x^2}{1+2x}$ ⟶ $f = x^2, \quad g = 1+2x$

$h(x)$

$$h' = \frac{f'g-fg'}{g^2} = \frac{2x(1+2x)-x^2(2)}{(2x+1)^2} = \frac{2x+4x^2-2x^2}{(\ )^2} = \frac{2x+2x^2}{(2x+1)^2}$$

$$= 2\left(\frac{x+x^2}{(2x+1)^2}\right) \Longrightarrow$$

$g = (2x+1)^2 \Rightarrow g' = 2(2x+1)(2)$ using 2.5 chain Rule!

$$h'' = \frac{f'g-fg'}{g^2} = 2\,\frac{(1+2x)(2x+1)^2-(x+x^2)(4(2x+1))}{(2x+1)^4}$$

$$= \frac{2\cancel{(2x+1)}\left[(1+2x)(2x+1)-(x+x^2)(4)\right]}{(2x+1)^{\cancel{4}\,3}}$$

$$= \frac{2\left[(2x+1)(2x+1)-(x+x^2)(4)\right]}{(2x+1)^3}$$

$$= \frac{2\left[4x^2+4x+1-4x-4x^2\right]}{(\ )^3} = \boxed{\frac{2}{(2x+1)^3}}\ !$$

$= h''$

72. If $h(2) = 4$ and $h'(2) = -3$, find

$$\frac{d}{dx}\left(\frac{h(x)}{x}\right)\bigg|_{x=2}$$

$f = h = h(x)$
$g = x$

$$= \frac{f'g - fg'}{g^2}\bigg|_{x=2} = \frac{h'(x)x - h(x)\cdot 1}{x^2}\bigg|_{x=2} = \frac{h'(2)(2) - h(2)}{2^2}$$

$$= \frac{-3(2)-4}{2^2} = \frac{-6-4}{4} = \frac{-10}{4} = \boxed{-\frac{5}{2} = \frac{d}{dx}\left[\frac{h(x)}{x}\right]\bigg|_{x=2}}$$

82. Find equations of the tangent lines to the curve

$$y = \frac{x-1}{x+1}$$

that are parallel to the line $x - 2y = 2$.

$m_{\parallel} = m$

m for $Ax + By = C$ is
$m = -\frac{A}{B}$ Bleah.

$m_{\parallel} = m$ parallel
$m_{\perp} = m$ perpendicular

$Ax + By = C \Rightarrow$
$By = -Ax + C \Rightarrow$
$y = \frac{-Ax+C}{B} = -\frac{A}{B}x + \frac{C}{B}$

$x - 2y = 2$
$-2y = -x + 2$
$y = \frac{-x+2}{-2} = \frac{-x}{-2} - 1$
$= \frac{1}{2}x - 1 \rightarrow \boxed{m = \frac{1}{2} = \text{Slope we want}}$

Our Job: Find $y' \stackrel{\text{SET}}{=} \frac{1}{2}$ & solve.

$$y = \frac{x-1}{x+1} \Rightarrow y' = \frac{1(x+1) - (x-1)(1)}{(x+1)^2} = \frac{x+1-x+1}{(x+1)^2} = \frac{2}{(x+1)^2} \stackrel{\text{SET}}{=} \frac{1}{2}$$

$\Rightarrow 4 = (x+1)^2 \Rightarrow$
$(x+1)^2 = 4 \Rightarrow$
$x+1 = \pm\sqrt{4} = \pm 2$
$\Rightarrow x = -1 \pm 2$; $1 = x$; $-3 = x$

$y = \frac{x-1}{x+1}$

$x = 1 \Rightarrow y(1) = \frac{1-1}{1+1} = \frac{0}{2} = 0 \rightarrow (x_1, y_1) = (1, 0)$

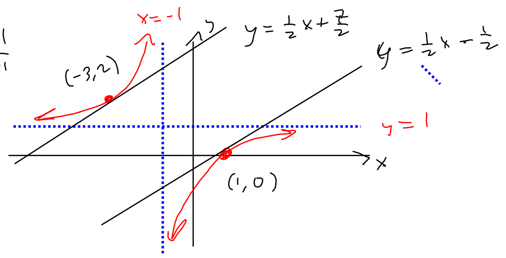
$y = \frac{1}{2}(x-1) + 0$
$\boxed{y = \frac{1}{2}x - \frac{1}{2}}$

$x = -3$: $y(-3) = \frac{-3-1}{-3+1} = \frac{-4}{-2} = 2 \Rightarrow (x_1, y_1) = (-3, 2)$

$y = \frac{1}{2}(x+3) + 2$
$= \frac{1}{2}x + \frac{3}{2} + \frac{4}{2} = \boxed{\frac{1}{2}x + \frac{7}{2} = y}$

Sketch of what's going on. $y = \frac{x-1}{x+1}$

This sketch was NOT asked for!



2.4

31. (a) Use the Quotient Rule to differentiate the function

$$f(x) = \frac{\tan x - 1}{\sec x}$$

(b) Simplify the expression for $f(x)$ by writing it in terms of $\sin x$ and $\cos x$, and then find $f'(x)$.

(c) Show that your answers to parts (a) and (b) are equivalent.

(a) $$f'(x) = \frac{\sec^2(x)\cancel{\sec(x)} - (\tan(x)-1)(\cancel{\sec(x)}\tan(x))}{\sec^{2}(x)}$$

1

$\tan^2(x)+1 = \sec^2(x)$!

Pythagorean Ident.!

$$= \frac{\sec^2(x) - \tan^2(x) + \tan(x)}{\sec(x)}$$

https://harryzaims.com/122/122-spring-18/tests-u-took/00-cheat-sheet.pdf

Lots of Trig Identities!

(a) $$= \frac{1+\tan(x)}{\sec(x)}$$ = what they want, but keep going

Part (c)

$$= \frac{1}{\sec(x)} + \frac{\tan(x)}{\sec(x)} = \cos(x) + \tan(x)\cos(x)$$

$$= \cos(x) + \frac{\sin(x)}{\cos(x)}\cos(x) = \cos(x) + \sin(x)$$!

(b) $$\frac{\tan(x)-1}{\sec(x)}$$

$$= \left(\frac{\sin(x)}{\cos(x)} - 1\right)(\cos(x)) = \sin(x) - \cos(x) \Longrightarrow$$

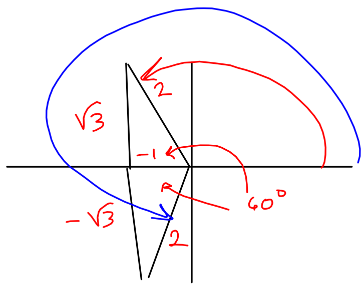
$$y' = \cos(x) - (-\sin(x)) = \cos(x) + \sin(x)$$!

33. For what values of x does the graph of $f(x) = x + 2\sin x$ have a horizontal tangent?

$f'(x) = 1 + 2\cos(x) \overset{\text{SET}}{=} 0$

$\Rightarrow 2\cos(x) = -1$

$\cos(x) = -\frac{1}{2}$



$180^\circ - 60^\circ = 120^\circ \left(\frac{\pi}{180^\circ}\right) = \frac{2\pi}{3}$

$180^\circ + 60^\circ = 240^\circ \left(\frac{\pi}{180^\circ}\right) = \frac{4\pi}{3}$

And this will keep happening over and over, so

$\frac{2\pi}{3} + 2\pi n,\ n \in \mathbb{Z}$

$\frac{4\pi}{3} + 2\pi n,\ n \in \mathbb{Z}$

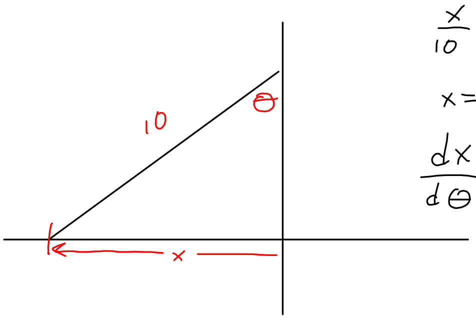
$\mathbb{Z} = \{x \mid x \text{ is an integer}\}$

$= \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$

37. A ladder 10 ft long rests against a vertical wall. Let θ be the angle between the top of the ladder and the wall and let x be the distance from the bottom of the ladder to the wall. If the bottom of the ladder slides away from the wall, how fast does x change with respect to θ when $\theta = \pi/3$?

2.5

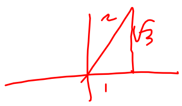
Later, we'll do a falling ladder where they tell you how fast the top is sliding down in ft/s.



$$\frac{x}{10} = \sin\theta$$

$$x = 10\sin\theta$$

$$\left.\frac{dx}{d\theta}\right|_{\theta=\frac{\pi}{3}} = \left.10\cos\theta\right|_{\theta=\frac{\pi}{3}} = 10\cos\left(\frac{\pi}{3}\right) = 10\left(\frac{1}{2}\right) = 5$$



$$\boxed{= 5\ \frac{\text{ft}}{\text{radian}}}$$

39–50 Find the limit.

39. $\lim\limits_{x\to 0} \dfrac{\sin 5x}{3x}$

41. $\lim\limits_{t\to 0} \dfrac{\tan 6t}{\sin 2t}$

2.5

#17. Differentiate $(x^2+1)^3(x^2+x+1)^5$

$h(x) = (x^2+1)^3(x^2+x+1)^5 = fg$

$h'(x) = f'g + fg' =$

$3(x^2+1)^2(2x)(x^2+x+1)^5 + (x^2+1)^3\left(5(x^2+x+1)^4(2x+1)\right)$

$= (x^2+1)^2(x^2+x+1)^4\left[3(2x)(x^2+x+1) + (x^2+1)(5)(2x+1)\right]$

$= (\quad)^2(\quad)^4\left[6x(x^2+x+1) + (5x^2+5)(2x+1)\right]$

$= (\quad)^2(\quad)^4\left[6x^3 + 6x^2 + 6 + 10x^3 + 5x^2 + 10x + 5\right]$

$= (x^2+1)^2(x^2+x+1)^4\left[16x^3 + 11x^2 + 10x + 11\right]$ is

what WebAssign Probably wants!

Ugh!

$$f(x)=\begin{cases} x^2\sin\left(\frac{\pi}{x}\right) & \text{if } x\neq 0 \\ 0 & \text{if } x=0 \end{cases}$$ is differentiable everywhere!

Clearly dif b^le away from $x=0$

What about @ $x=0$?

$$\lim_{h\to 0}\frac{f(0+h)-f(0)}{h} = \lim_{h\to 0}\frac{h^2\sin\left(\frac{\pi}{h}\right)-0}{h}$$

$$=\lim_{h\to 0} h\sin\left(\frac{\pi}{h}\right) \xrightarrow{h\to 0} 0$$

why?

$$-1 \le \sin\left(\frac{\pi}{h}\right) \le 1 \quad \text{if } h\neq 0$$

$$-h \le h\sin\frac{\pi}{h} \le h$$

$$\downarrow{\scriptstyle h\to 0} \quad \downarrow{\scriptstyle h\to 0} \quad \downarrow{\scriptstyle h\to 0}$$

$$0 \qquad 0 \qquad 0$$

By Squeeze.

$$r(x)=\begin{cases} x\sin\left(\frac{\pi}{x}\right) & \text{if } x\neq 0 \\ 0 & \text{if } x=0 \end{cases}$$ is continuous everywhere,

but Not differentiable @ $x=0$.