Test 2 is scheduled for the end of this week.

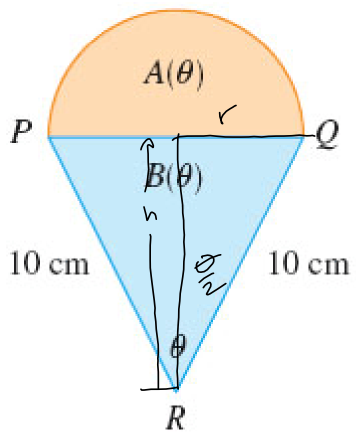
I'll open it up Friday and leave it open over the weekend.

§2.4 #23

23. 0/1 points SCalc8 2.4.056. [3354197

A semicircle with diameter PQ sits on an isosceles triangle PQR to form a region shaped like a two-dimensional ice-cream cone, as shown in the figure. If $A(\theta)$ is the area of the semicircle and $B(\theta)$ is the area of the triangle, find

$$\lim_{\theta \to 0^+} \frac{A(\theta)}{B(\theta)}.$$



$\cos\left(\frac{\theta}{2}\right) = \frac{h}{10} \Rightarrow h = 10\cos\frac{\theta}{2}$

$\sin\left(\frac{\theta}{2}\right) = \frac{r}{10} \Rightarrow r = 10\sin\left(\frac{\theta}{2}\right)$

Area of Δ is $\frac{1}{2}bh = rh$

$B = \frac{1}{2}rh = \frac{1}{2}\left(10\sin\left(\frac{\theta}{2}\right)\right)\left(10\cos\left(\frac{\theta}{2}\right)\right)$

$B(\theta) = \overset{100}{50}\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)$

$A(\theta) = \frac{1}{2}\left(\pi r^2\right) = \frac{1}{2}\pi\left(10\sin\frac{\theta}{2}\right)^2$

$= 50\pi\sin^2\left(\frac{\theta}{2}\right)$

$\frac{A(\theta)}{B(\theta)} = \frac{50\pi\sin^2\left(\frac{\theta}{2}\right)}{\underset{100}{50}\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)} = \frac{\pi\sin\left(\frac{\theta}{2}\right)}{2\cos\left(\frac{\theta}{2}\right)}$

$\theta \to 0^+ \Rightarrow \boxed{0}$

Section 2.8: Related Rates.

It's all about being able to

#1. Model with a formula

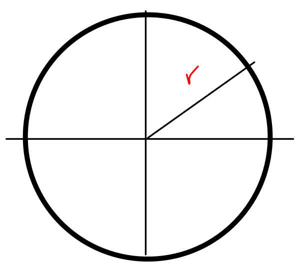
#2. Differentiate implicitly, using the Chain Rule.

#3. Putting things together.

1. Question Details SCalc8 2.8.002. [3354270]

(a) If *A* is the area of a circle with radius *r* and the circle expands as time passes, find *dA*/*dt* in terms of *dr*/*dt*.

(b) Suppose oil spills from a ruptured tanker and spreads in a circular pattern. If the radius of the oil spill increases at a constant rate of 1 m/s, how fast is the area of the spill increasing when the radius is 24 m?



Picking up with something almost the same as the exercise where we left off on Friday. (See notes and (edited) video).

$A = \pi r^2$

A = Area of oil slick as function of

r = radius of oil slick, which is func of

t = time.

Given $\boxed{\frac{dr}{dt} = 1 \ m/s}$

Want $\left.\frac{dA}{dt}\right|_{r=24\text{ m}} = A'(24)$

$\frac{d}{dt}\left[A = \pi r^2\right]$

Chain Rule!

Part (a)

$$\boxed{\frac{dA}{dt} = 2\pi r \cdot \frac{dr}{dt}}$$

$\frac{d}{dt}\left[f(t)^7\right] = 7f(t)^6 f'(t)$

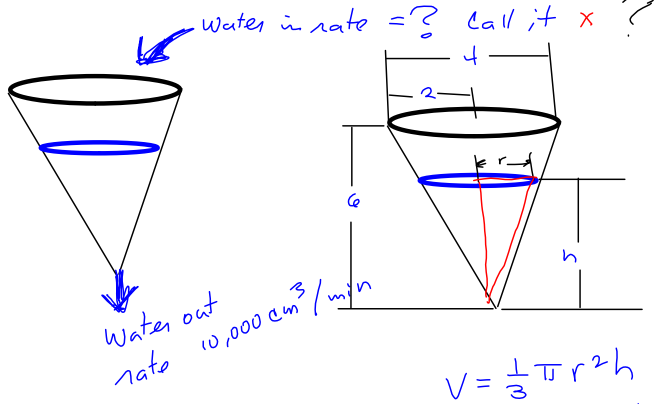
$\left.\frac{dA}{dt}\right|_{r=24\text{ m}} = 2\pi(24)(1) = \boxed{48\pi \ \frac{m^2}{s}}$

Part (b)

15. A street light is mounted at the top of a 15-ft-tall pole. A man 6 ft tall walks away from the pole with a speed of 5 ft/s along a straight path. How fast is the tip of his shadow moving when he is 40 ft from the pole?

25. Water is leaking out of an inverted conical tank at a rate of 10,000 cm^3/min at the same time that water is being pumped into the tank at a constant rate. The tank has height 6 m and the diameter at the top is 4 m. If the water level is rising at a rate of 20 cm/min when the height of the water is 2 m, find the rate at which water is being pumped into the tank.

Water in rate = ? Call it x ?



$\left.\frac{dh}{dt}\right|_{h=2m=200cm} = \frac{20\,cm}{min}$

$V = \frac{1}{3}\pi r^2 h$

r = radius at top of H_2O (cm)

h = " of water (cm)

By similar triangles

$\frac{h}{r} = \frac{6}{2}$

$\boxed{h = 3r}$

V = Volume in cubic cm.

t = time in minutes

volume of water is $\frac{1}{3}\pi r^3 h = V$

Want x = rate of filling.

$\frac{dV}{dt} = -10{,}000 \frac{cm^3}{min} + x$

Want x = rate of filling $\frac{cm^3}{min}$ when h = 2 m

$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi r^2 (3r) = \pi r^3 \Longrightarrow$

$\frac{dV}{dt} = 3\pi r^2 \cdot \frac{dr}{dt} = 3\pi \left(\frac{2}{3}\right)^2 \cdot \left(\frac{1}{3}\frac{dh}{dt}\right) = 3\pi\left(\frac{4}{9}\right)\left(\frac{1}{3}\right)(20)$

$= \boxed{\frac{80\pi}{9} \frac{cm^3}{min}}$

we know $h = 3r$

so $\frac{dh}{dt} = 3\frac{dr}{dt}$

$\Longrightarrow \frac{dr}{dt} = \frac{1}{3}\frac{dh}{dt}$

$h = 3r$

$2 = 3r$

$\frac{2}{3} = r$ when h = 2

$\frac{80\pi}{9}\frac{cm^3}{min} = \text{NET} = x - 10{,}000 \frac{cm^3}{min}$

$\Longrightarrow x = \left(\frac{80\pi}{9} + 10{,}000\right) \frac{cm^3}{min}$

Re-work, but THIS time, got rid of r, instead of h.

$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi\left(\frac{1}{3}h\right)^2 h = \frac{1}{27}\pi h^3$

$\Longrightarrow \frac{dV}{dt} = \frac{3}{27}\pi h^2 \frac{dh}{dt} = \frac{1}{9}\pi (2)^2 (20) = \frac{80\pi}{9} \frac{cm^3}{min}$

(Net rate of volume's increase)

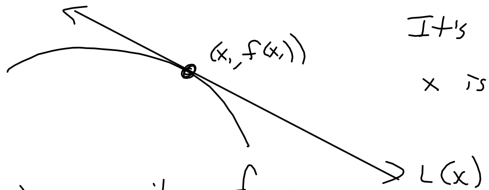
§2.9 Linear Approximations & Differentials.

Recall: $y = m(x-x_1) + y_1$ is the eq'n of the line thru (x_1, y_1) with slope m.

TANGENT LINE: $m = f'(x_1)$ & $y_1 = f(x_1)$

$y = f'(x_1)(x-x_1) + f(x_1)$, which the book re-writes as $L(x) = f(x_1) + f'(x_1)(x-x_1)$

This is the "Linearization" or "Linear Approximation" for $f(x)$ @ $(x_1, f(x_1))$



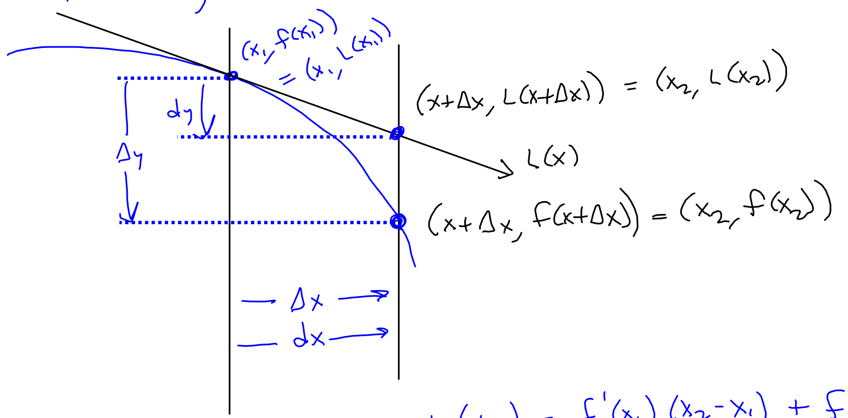
It's close to $f(x)$ when x is close to x_1.

We've done quite a few.

We haven't done differentials in this discussion.

$dx = \Delta x$ = small change in x

Δy = Actual Change in $f(x) \approx dy$ = tangent line's idea of the change in $f(x)$.



$L(x_2) = f'(x_1)(x_2 - x_1) + f(x_1)$

$$L(x) = f'(x_1)\underbrace{(x-x_1)} + f(x_1)$$

$$dy = \boxed{f'(x_1)\Delta x = f'(x_1)\,dx} \approx \Delta y$$

often easier to calculate than $f(x+\Delta x) - f(x) = \Delta y$

$$\underbrace{L(x_2) - f(x_1)}_{L(x_2) - L(x_1)} = f'(x_1)(x_2 - x_1)$$

$L(x_2) - L(x_1)$, since $L(x_1) = f(x_1)$.

$$\Delta y = f(x_2) - f(x_1) \approx L(x_2) - L(x_1) = f'(x_1)\Delta x$$

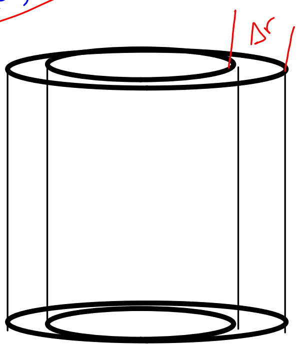
35. (a) Use differentials to find a formula for the approximate volume of a thin cylindrical shell with height h, inner radius r, and thickness Δr.

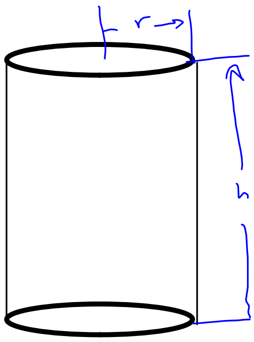
(b) What is the error involved in using the formula from part (a)?

(c) ~~Relative Error?~~

(d) ~~Percent Error?~~

Volume of right circular cylinder.





$V = \pi r^2 h$

Want: ΔV

$\approx 2\pi r \, dr \, h$

h = constant, like "2" or "π"

ΔV = Volume of outer cylinder − Volume of inner cylinder

$= \pi (r+\Delta r)^2 h - \pi (r)^2 h$

$= \pi h \left[(r+\Delta r)^2 - r^2 \right] = \pi h \left[r^2 + 2r\Delta r + (\Delta r)^2 - r^2 \right]$

$= \pi h \left[2r\Delta r + (\Delta r)^2 \right]$ = Actual volume of the wall of cylinder.

But $dV = 2\pi r \, dr \, h = 2\pi r \, \Delta r \, h$

The error is:

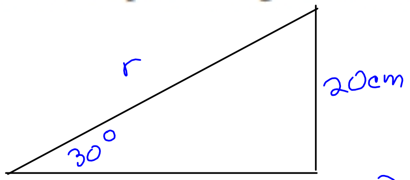
$2\pi r \Delta r h + \pi h (\Delta r)^2 - 2\pi r dr h = \boxed{\pi h (\Delta r)^2 = \text{error}}$

36. One side of a right triangle is known to be 20 cm long and the opposite angle is measured as 30°, with a possible error of ±1°.

(a) Use differentials to estimate the error in computing the length of the hypotenuse.

(b) What is the percentage error?

$30^\circ \pm 1^\circ$ OR $\frac{\pi}{6} \pm \frac{\pi}{180}$



$\frac{20}{r} = \sin 30^\circ$

$r = \frac{20}{\sin 30^\circ} = \frac{20}{\frac{1}{2}} = 40$

Do these in radians, to be safe.

r = length of hypotenuse (in cm)

$\boxed{\theta = \text{angle in radians}}$

$r = \frac{20}{\sin(\frac{\pi}{6})}$

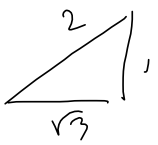
$\frac{d}{d\theta}[\csc\theta] = -\csc\theta\cot\theta$

$r = \frac{20}{\sin\theta} = 20\csc\theta$

$\Delta r \approx dr = -20\csc\theta\cot\theta\, d\theta = -20\csc\left(\frac{\pi}{6}\right)\cot\left(\frac{\pi}{6}\right)\left(\pm\frac{\pi}{180}\right)$

$\theta = \frac{\pi}{6}$

$d\theta = \frac{\pi}{180^\circ}$



$= \mp\frac{\pi}{90}(2)(\sqrt{3}) = \mp\frac{\sqrt{3}\pi}{45}$

(a) $\boxed{\approx \pm .1209199577 \text{ cm}}$

(b) % error:

(Relative Error)(100%)

$\left(\frac{\Delta r}{r}\right)100\% \approx \frac{\pm .1209199577}{40} \approx \pm$

$\boxed{\pm .3022998942\ \%}$