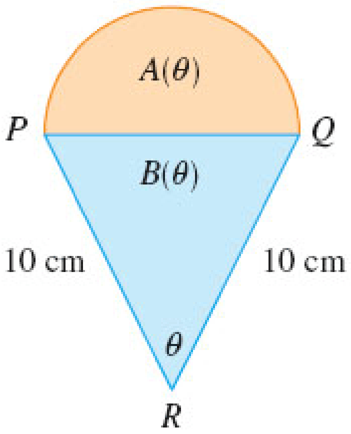
§ 2.4 #23

23. 0/1 points SCalc8 2.4.056. [3354197

A semicircle with diameter *PQ* sits on an isosceles triangle *PQR* to form a region shaped like a two-dimensional ice-cream cone, as shown in the figure. If $A(\theta)$ is the area of the semicircle and $B(\theta)$ is the area of the triangle, find

$$\lim_{\theta \to 0^+} \frac{A(\theta)}{B(\theta)}.$$

0



10. A particle moves with position function

$$s = t^4 - 4t^3 - 20t^2 + 20t \qquad t \geqslant 0$$

(a) At what time does the particle have a velocity of 20 m/s?

(b) At what time is the acceleration 0? What is the significance of this value of t?

(a) s = position in meters as a function of t = time in seconds.

$s = t^4 - 4t^3 - 20t^2 + 20t = s(t) =$ "s of t" $\Rightarrow$

$s'(t) = 4t^3 - 12t^2 - 40t + 20 \overset{SET}{=} 20$

$\Rightarrow 4t^3 - 12t^2 - 40t = 0$

$\Rightarrow 4t(t^2 - 3t - 10) = 4t(t-5)(t+2) = 0 \Rightarrow$

$t \in \{-2, 0, 5\}$ but $t \neq -2$, b/c $t \geq 0$.

so $\boxed{t \in \{0, 5\}.}$

(b) $s''(t) = 0$; $s''(t) = 12t^2 - 24t - 40 \overset{SET}{=} 0 \Rightarrow$

$a=3$, $b=-6$, $c=-10$

$4(3t^2 - 6t - 10) = 0$

$\Rightarrow b^2 - 4ac = (-6)^2 - 4(3)(-10)$

$= 36 + 120 = 156$

-30

$-6 = -7 + 1$ $\quad -7$

$= -8 + 2$ $\quad -16$

$= -9 + 3$ $\quad -27$

$= -10 + 4$ $\quad -40$ } Looking for -30

$b^2 - 4ac$ isn't a perfect square, silly!

$2 \,|\, 156$, $2 \,|\, 78$, $3 \,|\, 39$, 13

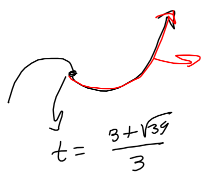
$t = \dfrac{6 \pm 2\sqrt{39}}{6}$

$\dfrac{6 + 2\sqrt{39}}{6} = \dfrac{3 + \sqrt{39}}{3}$

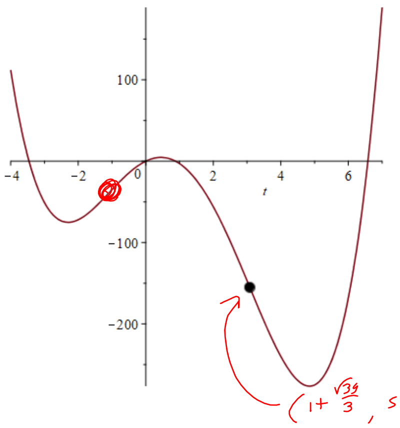
$\dfrac{6 - 2\sqrt{39}}{3} < 0$ Not in domain.

So $\boxed{t = \dfrac{3 + \sqrt{39}}{3}}$

It's when we have $s'' = 0$.



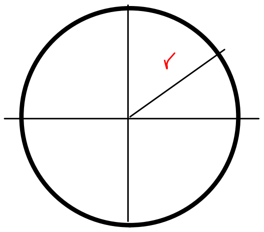
eventually does this, because it's a t^4 situation.



$\left(1 + \frac{\sqrt{39}}{3}, s\left(\frac{3+\sqrt{39}}{3}\right)\right)$ is where slope stops decreasing

14. A stone is dropped into a lake, creating a circular ripple that travels outward at a speed of 60 cm/s. Find the rate at which the area within the circle is increasing after (a) 1 s, (b) 3 s, and (c) 5 s. What can you conclude?

Related Rates, Later



$\frac{dr}{dt} = 60 \text{ cm/sec}$

r = radius of ripple, in cm.

$A = \pi r^2$ = Area as function of r, in cm^2

$\frac{d}{dt}\left[A = \pi r^2\right]$

$\frac{dA}{dt} = \pi\left[2 r r'(t)\right] = \pi\left[2r \frac{dr}{dt}\right]$

$\boxed{\frac{dr}{dt} = 60}$

Want $\left.\frac{dA}{dt}\right|_{t=1,3,5}$
a b c

$r = r_0 + \frac{dr}{dt} t$

$= \frac{dr}{dt} t = 60t$

$\left.\frac{dA}{dt}\right|_{t=1} = \pi(2 \cdot 60t)(60)$

$= 720\pi t = \boxed{720\pi \frac{cm^2}{s}}$

$\left.\frac{dA}{dt}\right|_{t=} = \pi(2 \cdot 60t)(60)\Big|_{t=3}$

$= 720\pi t\Big|_{t=3}$

$= 2160\pi \frac{cm^2}{sec}$

$\left.\frac{dA}{dt}\right|_{t=5} = \ldots = 720\pi(5) = 3600\pi \frac{cm^2}{s}$

It looks linear!

But the area is expanding as the square of time!

$\frac{dr}{dt} = 60$

$r = 60t$

Lucas
Mayla
Cacie

$Area = \pi r^2$

$A = A(r) = A(r(t)) = \pi(60t)^2$

$\frac{dA}{dt} = 2\pi(60t)(60) = 720\pi t$

$A = A(r) = \pi r(t)^2$

$\frac{dA}{dt} = (2\pi r(t))\frac{dr}{dt} = 2\pi(60t)(60)$