Chain Rule

Newton

$$\frac{d}{dx}\left[f(g(x))\right] = f'(g)\cdot g'(x)$$

$$\frac{d\left[f(g(x))\right]}{dx} = \frac{df}{dg}\cdot\frac{dg}{dx}$$

Liebniz

$$\frac{d}{dx}\left[\cos(\pi x - 7)\right] = \left(-\sin(\pi x - 7)\right)\cdot \pi$$

Generalized power Rule

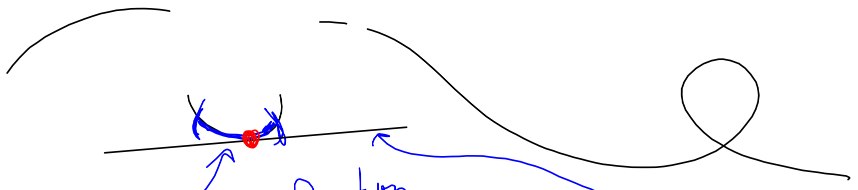
$$\frac{d}{dx}\left[(f(x))^n\right] = n\left(f(x)\right)^{n-1}\cdot f'(x)$$

$$\frac{d}{dx}\left[(\sin(x))^5\right] = 5\left(\sin(x)\right)^4\cdot \cos(x)$$

We start with some Section 2.5 exercises, back-track to previous sections, and then a pep talk on Section 2.6.

Some §2.6 questions

§ 2.6 Pep Talk. Not ALL relations/curves are functions, but behave "locally" as functions.



Like a function over this small piece

So it makes sense to talk about its slope there, or even to build a tangent line to it, there

We need the chain rule, in a big way.
We assume y is a function of x over short distances.

$$\frac{d}{dx}\left[(\sin(x))^2\right] = 2(\sin(x))^{2-1}\cdot\cos(x)$$

$$\frac{d}{dx}\left[(f(x))^2\right] = 2f(x)^{2-1}\cdot f'(x) = 2f(x)\,f'(x)$$

§2.6 #11

11. 0/4 points SCalc8 2.6.034. [3354466]

(a) The curve with equation $y^2 = x^3 + 3x^2$ is called the **Tschirnhausen cubic**. Find an equation of the tangent line to this curve at the point (1, 2).

$y =$ $\frac{9}{4}x - \frac{1}{4}$

$\Rightarrow y = \pm\sqrt{x^3+3x^2}$ Not a function!
We can still talk about y' in a meaningful way.

$$\frac{d}{dx}\left[y^2\right] = 2yy'$$

Differentiate Both Sides of the equation with respect to x.

$$y^2 = x^3 + 3x^2$$

$$\frac{d}{dx}\left(y^2 = x^3 + 3x^2\right)$$

$$2yy' = 3x^2 + 6x$$

$$y' = \frac{3x^2 + 6x}{2y}$$

Now, $(x, y) = (1, 2)$:

$$\Longrightarrow \left. y' \right|_{(x,y)=(1,2)} = \frac{3(1)^2 + 6(1)}{2(2)} = \frac{3+6}{4} = \boxed{\frac{9}{4} = m_{tan}}$$

$$y = m(x - x_1) + y_1$$

$$\boxed{y = \frac{9}{4}(x-1) + 2} = \frac{9}{4}x - \frac{9}{4} + 2\left(\frac{4}{4}\right) = \frac{9}{4}x - \frac{9}{4} + \frac{8}{4}$$

$$y = \frac{9}{4}x - \frac{1}{4}$$

Perfect for homework (written) or hand-written test.

Tangent Line:

$$y = f'(x_1)(x - x_1) + f(x_1)$$

This Page is the "Grunt Work" that went into the (fairly) nice write-up on the page after.

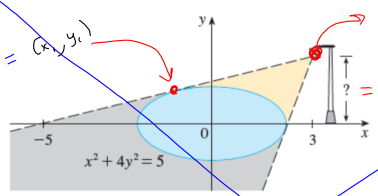
I think more was learned from seeing some (many) false moves than in seeing one video where I get the right answer without any flailing around. Students NEED to understand that there's going to be a lot of flailing around when they're problem-solving, and that's.... OK.

17. 0/1 points SCalc8 2.6.062.

The figure shows a lamp located three units to the right of the y-axis and a shadow created by the elliptical region $x^2 + 4y^2 \le 5$. If the point $(-5, 0)$ is on the edge of the shadow, how far above the x-axis is the lamp located?

2 units

$(a,b) = (x_1, y_1)$



(x_2, y_2) want $y_2 = ?$

$? =$ what we want.

$(-5,0)$ & $(3, y_2)$ on the tangent line.

Let's do y': Boundary

$$x^2 + 4y^2 = 5$$

$$\frac{d}{dx}\left(x^2 + 4y^2 = 5\right)$$

$$2x + 8yy' = 0$$

$$8yy' = -2x$$

$$\boxed{y' = \frac{-2x}{y}}$$

Basic Algebra, mills. $\frac{-2x}{8y} = -\frac{x}{4y}$

we want the x & y.

We know $(-5,0)$ & $(3, y_2)$ on the tangent line.

$$m_{tan} = \frac{y_2}{3-(-5)} = \frac{y_2}{8} \ldots \text{ So?}$$

So?

$$y' = m_{tan} = -\frac{x_1}{4y_1} = -\frac{a}{4b} = \frac{y_2}{8} = \frac{b}{a+5}$$

Since $(-5,0)$ & (a,b) are on the line,

$$m = \frac{b-0}{a+5} = \frac{b}{a+5}$$

$$\text{So, } -\frac{a}{4b} = \frac{b}{a+5} \Rightarrow -a(a+5) = 4b^2$$

$$-a^2 - 5a = 4b^2$$

Skipping a step (showing off or being in a hurry) messed me up!

$$\Rightarrow a^2 + 4b^2 = 5a \quad \text{But}$$

$x^2 + 4y^2 = 5$ on the boundary!

$$\Rightarrow a^2 + 4b^2 = -5 \Rightarrow \boxed{a = 1}!$$

$\boxed{a = -1}$

Doesn't match the pic.

$$a^2 + 4b^2 = 1 + 4b^2 = 5 \Rightarrow$$

$$4b^2 = 4$$

$$b^2 = 1$$

$$b = \pm 1 \Rightarrow b = -1 \text{ by picture!}$$

$+1$

$(a,b) = ($

So, $(a,b) = (-1, 1)$

$$\text{\& } y' = m_{tan} = -\frac{a}{4b} = \frac{-(-1)}{4(1)} = \frac{1}{4}$$

Tan. Line is $y = \frac{1}{4}(x+1) + 1$

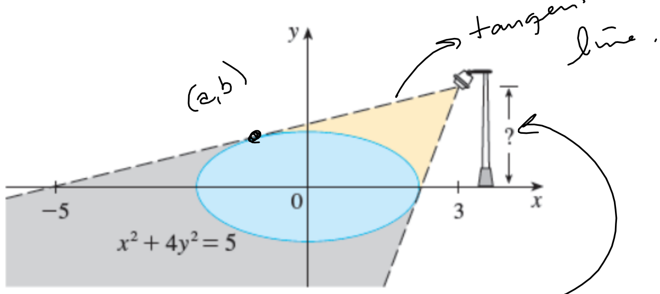
Find y-value for $x = 3$:

$$y_2 = \frac{1}{4}(3+1) + 1 = \frac{1}{4}(4) + 1 = 2 = \text{height!}$$

17. 0/1 points SCalc8 2.6.062.

The figure shows a lamp located three units to the right of the y-axis and a shadow created by the elliptical region $x^2 + 4y^2 \le 5$. If the point $(-5, 0)$ is on the edge of the shadow, how far above the x-axis is the lamp located?

✗ 2 units



we want to know "?"

It's the point on that tangent line corresponding to

$x = 3$.

$m_{tan} = \dfrac{b-0}{a-(-5)} = \dfrac{b}{a+5}$, using $m = \dfrac{y_2-y_1}{x_2-x_1}$, &

$(x_1, y_1) = (-5, 0)$, $(x_2, y_2) = (a, b)$

Now, we have another expression for slope @ (a,b)

m_{tan} : $x^2 + 4y^2 = 5$ (on the boundary)

$\Longrightarrow 2x + 8yy' = 0$, differentiating implicitly w.r.t. x

$\Longrightarrow 8yy' = -2x$

$\Longrightarrow y' = \dfrac{-2x}{8y} = -\dfrac{x}{4y}$. Plug in $(a,b) \Longrightarrow$

$$m_{tan} = -\frac{a}{4b} = \frac{b}{a+5}$$

$$\Rightarrow \quad -a(a+5) = 4b^2$$

$$-a^2 - 5a = 4b^2$$

$$+a^2 \quad = \quad + a^2$$

$$-5a = a^2 + 4b^2 \qquad \text{But } a^2+4b^2 = 5!$$

$$\Rightarrow \quad -5a = 5$$

$$\Rightarrow \quad \boxed{a = -1}$$

$a^2 + 4b^2 = 5$ it's on the ellipse!

$$(-1)^2 + 4b^2 = 5$$

$$1 + 4b^2 = 5$$

$$4b^2 = 4$$

$$b^2 = 1$$

$$b = \pm 1 \Rightarrow \boxed{b = +1} \text{ by picture.}$$

$$m = \frac{b-0}{a-(-5)} = \frac{b}{a+5} = \frac{1}{-1+5} = \frac{1}{4} = m_{tan} \text{ @ } (a,b) = (-1,1)$$

$$y = \frac{1}{4}(x+1) + 1$$

plug in $x=3$ to find height of light:

$$y = \frac{1}{4}(3+1) + 1 = \frac{1}{4}(4) + 1 = \boxed{2 \text{ units}}$$

1. 0/2 points — SCalc8 2.5.001. [335440

Write the composite function in the form $f(g(x))$. [Identify the inner function $u = g(x)$ and the outer function $y = f(u)$.]

$$y = \sqrt[3]{1 + 7x}$$

$(g(x), f(u)) = \Big($ [redacted]

Find the derivative dy/dx.

$\frac{dy}{dx} =$ [redacted]

$g(x) = 1+7x$

$f(u)$ (or $f(g)$)

$= \sqrt[3]{u}$ (or $\sqrt[3]{g}$)

3. 0/1 points

Find the derivative of the function.

$$F(x) = (3x^6 + 2x^3)^4$$

$F'(x) =$ [redacted]

21. 0/2 points SCalc8 2.5.078. [335

A model for the length of daylight (in hours) in Philadelphia on the tth day of the year is

$$L(t) = 12 + 2.8 \sin\left[\frac{2\pi}{365}(t - 80)\right].$$

Use this model to compare how the number of hours of daylight is increasing in Philadelphia on March 21 and May 21. (Assume there are 365 days in a year. Round your answers to four decimal places.)

March 21 $L'(t) =$ ✗

May 21 $L'(t) =$ ✗

20. 0/2 points SCalc8 2.5.077. [3354362]

A Cepheid variable star is a star whose brightness alternately increases and decreases. For a certain star, the interval between times of maximum brightness is 5.7 days. The average brightness of this star is 2.0 and its brightness changes by ±0.45. In view of these data, the brightness of the star at time t, where t is measured in days, has been modeled by the function

$$B(t) = 2.0 + 0.45 \sin\left(\frac{2\pi t}{5.7}\right).$$

(a) Find the rate of change of the brightness after t days.

$\frac{dB}{dt} =$ ✗

(b) Find, correct to two decimal places, the rate of increase after five days.

$\frac{dB}{dt} =$

16. 0/2 points SCalc8 2.5.059. [3694806]

Find all points on the graph of the function $f(x) = 2\sin(x) + \sin^2(x)$ at which the tangent line is horizontal. (Use n as your arbitrary integer.)

$(x, y) = ($

$(x, y) = ($

15. 0/2 points

(a) If $f(x) = x\sqrt{10 - x^2}$, find $f'(x)$.

$f'(x) =$ $\frac{10-2x^2}{\sqrt{10-x^2}}$

$f(x) = x\sqrt{10-x^2} = x(10-x^2)^{\frac{1}{2}}$

Product Rule & chain Rule

$h = x,\ g = (10-x^2)^{\frac{1}{2}} \Rightarrow g' = \frac{1}{2}(10-x^2)^{-\frac{1}{2}}(-2x)$

$(hg)' = h'g + hg' = 1g + xg'$

& g' takes Chain Rule:

$g = j(k(x))$ $\quad k(x) = 10 - x^2$

$j(u) = u^{1/2}$

$j(k) = k^{\frac{1}{2}} \Rightarrow j'(k) = \frac{1}{2}k^{\frac{1}{2}-1} = \frac{1}{2}k^{-\frac{1}{2}} = \frac{1}{2}(10-x^2)^{-\frac{1}{2}}$

and $k'(x) = -2x$

$(j(k(x)))' = (j'(k))(k'(x))$

$= \frac{1}{2}(10-x^2)^{-\frac{1}{2}}(-2x)$

$$\frac{d}{dx}\left[x(10-x^2)^{\frac{1}{2}}\right] = \overset{f'}{1}\overset{g}{(10-x^2)^{\frac{1}{2}}} + \boxed{\overset{f}{x}\overset{g'}{\left(\tfrac{1}{2}(10-x^2)\right)^{-\frac{1}{2}}(-2x)}}$$

$\hookrightarrow x\left(\frac{1}{2}\right)(10-x^2)^{-1/2}(-2x)$

$= \sqrt{10-x^2} - \cancel{2}x^2\left(\frac{1}{\cancel{2}}\right)(10-x^2)^{-\frac{1}{2}}$

$= \sqrt{10-x^2} - \frac{x^2}{(10-x^2)^{1/2}} = \sqrt{10-x^2} - \frac{x^2}{\sqrt{10-x^2}}$

$= \left(\frac{\sqrt{10-x^2}}{1}\right)\left(\frac{\sqrt{10-x^2}}{\sqrt{10-x^2}}\right) - \frac{x^2}{\sqrt{10-x^2}}$

$= \frac{10-x^2-x^2}{\sqrt{10-x^2}} = \frac{10-2x^2}{\sqrt{10-x^2}} = \frac{2(5-x^2)}{\sqrt{10-x^2}}$

WebAssign.

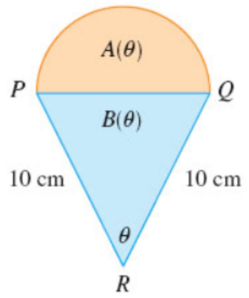
Some 2.4 Examples

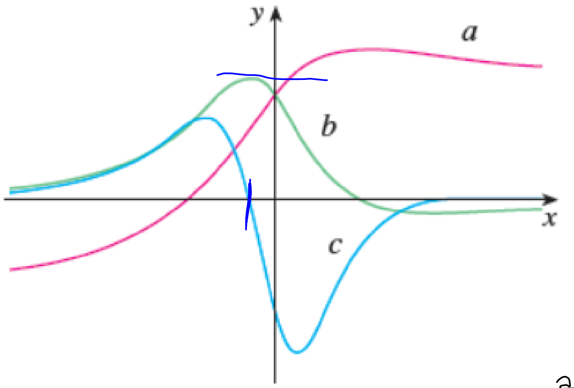
23. 0/1 points SCalc8 2.4.056. [3354197

A semicircle with diameter *PQ* sits on an isosceles triangle *PQR* to form a region shaped like a two-dimensional ice-cream cone, as shown in the figure. If $A(\theta)$ is the area of the semicircle and $B(\theta)$ is the area of the triangle, find

$$\lim_{\theta \to 0^+} \frac{A(\theta)}{B(\theta)}.$$

0





f, f', f" ?

(f''' = "jerk")

I'm thinkin'

c = b' b/c c = max where

b' = 0

a = f, b = f', c = f"