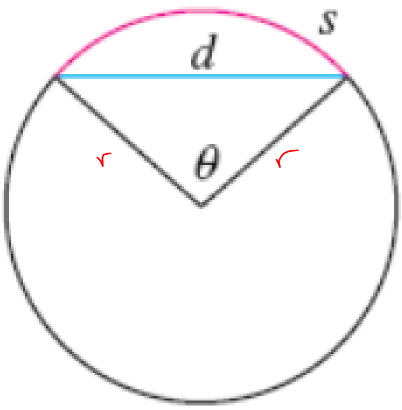
Section 2.4 #24 (Last one. It's pretty tricky.)

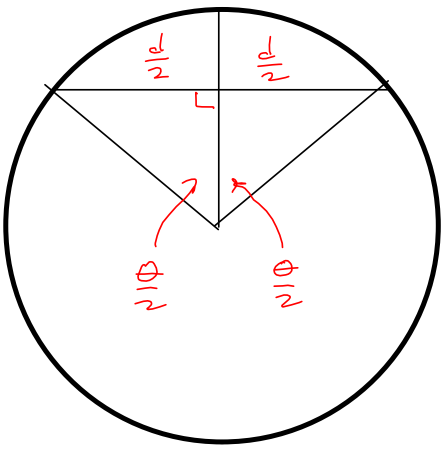
0/1 points SCalc8

The figure shows a circular arc of length s and a chord of length d, both subtended by a central angle θ. Find

$\lim_{\theta \to 0^+} \frac{s}{d}$.

$\sin\left(\frac{\theta}{2}\right) = \frac{\frac{d}{2}}{r} = \frac{d}{2r} \Rightarrow 2r\sin\theta = d$





$d = 2r\sin\left(\frac{\theta}{2}\right)$

$s = \text{arc length} = r\theta$

$\Rightarrow \frac{s}{d} = \frac{r\theta}{2r\sin\left(\frac{\theta}{2}\right)}$

$= \frac{\theta}{2\sin\left(\frac{\theta}{2}\right)} = \frac{2\left(\frac{\theta}{2}\right)}{2\sin\left(\frac{\theta}{2}\right)} = \frac{\frac{\theta}{2}}{\sin\left(\frac{\theta}{2}\right)} = \frac{u}{\sin(u)}$, where

$u = \frac{\theta}{2}$. Note $\theta \to 0 \Rightarrow \frac{\theta}{2} = u \to 0$

$\Rightarrow \lim_{u \to 0^+} \frac{u}{\sin(u)} = 1$, from $\lim_{h \to 0} \left(\frac{\sin(h)}{h}\right) = 0$.

2.1, 2.2 Lot of conceptual stuff & I like to give you a cheat to check the work using the tools that come later & the 2.1 & 2.2 are supposed to motivate.

Rules for derivatives:

Definition $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = f'(x)$

2.3

Power Rule: $\frac{d}{dx}[x^n] = nx^{n-1}$ (if $n \neq 0$)

$n=0 \Rightarrow x^n = x^0 = 1$ has slope $m=0$

Product Rule: $(fg)' = f'g + fg'$

Quotient Rule: $\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$

Recall §2.4:

$\frac{d}{dx}[\sin(x)] = \cos(x)$ $\frac{d}{dx}[\csc(x)] = -\csc(x)\cot(x)$

$\frac{d}{dx}[\cos(x)] = -\sin(x)$ $\frac{d}{dx}[\sec(x)] = \sec(x)\tan(x)$

$\frac{d}{dx}[\tan(x)] = \sec^2(x)$ $\frac{d}{dx}[\cot(x)] = -\csc^2(x)$

§2.5 $\big(f(g(x))\big)' = f'(g(x)) \cdot g'(x)$

$$\frac{d[f(g(x))]}{dx} = \frac{d[f(g(x))]}{d[g(x)]} \cdot \frac{d[g(x)]}{dx}$$

$$\frac{d}{dx}[\sin(x^2-3x)] = \cos(x^2-3x) \cdot (2x-3)$$
$$= (2x-3)\cos(x^2-3x)$$

I wanted this as a cheat for when they hit $\frac{d}{dx}[\sqrt{3x-2}]$ by the limit definition

$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\sqrt{3(x+h)-2} - \sqrt{3x-2}}{h}$ is painful

But the Chain Rule gives you an answer to check against.

§2.4

21. 0/1 points

Find the limit.

$$\lim_{x\to 6} \frac{\sin(x-6)}{x^2+x-42}$$

x^2+x-42

$(x+7)(x-6)$

$$= \lim_{x\to 6} \frac{\sin(x-6)}{(x-6)(x+7)} = \lim_{x\to 6}\left(\frac{\sin(x-6)}{x-6}\cdot\frac{1}{x+7}\right)$$

$$= \lim(\text{product}) = \text{product of limits}^{*} = \boxed{\lim_{x\to 6}\frac{\sin(x-6)}{x-6}}\ \lim_{x\to 6}\left(\frac{1}{x+7}\right)$$

$x\to 6 \implies u\to 0$

$u+6=x$ is true, but no help

$\lim_{u\to 0}\left(\frac{\sin(u)}{u}\right)=1$!

using $u=x-6$

$$= (1)\left(\frac{1}{6+7}\right) = \boxed{\frac{1}{13} = \lim_{x\to 6}\frac{\sin(x-6)}{x^2+x-42}}$$

* $\lim(fg) = (\lim f)(\lim g)$ if $\lim f$ & $\lim g$ exist, separately.

Basic 2.4 Skills

$$\frac{d}{dx}\left[\sin(x)\tan(x)\right]$$

Trig Derivatives & Product Rule

$$= (fg)' = f'g + fg' = \boxed{\cos(x)\tan(x) + \sin(x)\sec^2(x)}$$

FINE FOR HAND-WRITTEN TEST

$f = \sin(x)$ $\quad g = \tan(x)$

$f' = \cos(x)$ $\quad g' = \sec^2(x)$

$$= \cancel{\cos}(x)\left(\frac{\sin(x)}{\cancel{\cos}(x)}\right) + \sin(x)\left(\frac{1}{\cos^2(x)}\right)$$

$$= \sin(x) + \frac{\sin(x)}{\cos(x)} \cdot \frac{1}{\cos(x)} = \sin(x) + \tan(x)\sec(x)$$

$$= \sin(x)\left[1 + \sec^2(x)\right]$$ is another possibility.

It depends on what you want to do with it!

§2.3

26. 0/1 points

Find a second-degree polynomial P such that $P(2) = 13$, $P'(2) = 10$, and $P''(2) = 6$.

$P(x) =$ ✗ $3x^2 - 2x + 5$

$P(x) = ax^2 + bx + c$

$P(2) = 13 : a(2)^2 + b(2) + c = 13$

$4a + 2b + c = 13$

$P'(x) = 2ax^1 + b$

$P'(2) = 10 = 2a(2) + b = 4a + b = 10$

$P''(x) = 2a$

$P''(2) = 6 = 2a = 6$

$\boxed{a = 3}$

$4(3) + b = 10$

$12 + b = 10$

$\boxed{b = -2}$

$4a + 2b + c = 4(3) + 2(-2) + c = 13$

$12 - 4 + c = 13$

$8 + c = 13$

$\boxed{c = 5}$

$P(x) = ax^2 + bx + c = \boxed{3x^2 - 2x + 5}$

0/1 points

For what values of x does the graph of $f(x)$ have a horizontal tangent? (Enter your answers as a c

$f(x) = x^3 + 3x^2 + x + 5$

$x =$ $\frac{1}{6}\left(-6-2\sqrt{6}\right), \frac{1}{6}\left(2\sqrt{6}-6\right)$

$f' = 0$

$f'(x) = 3x^2 + 6x^1 + 1 \overset{SET}{=} 0$

$a = 3, b = 6, c = 1$

$b^2 - 4ac = 6^2 - 4(3)(1) = 36 - 12 = 24$

$2 \,|\, 24$
$2 \,|\, 12$
$2 \,|\, 6$
3

$\sqrt{24} = 2\sqrt{6}$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \dfrac{-6 \pm 2\sqrt{6}}{2(3)} = \boxed{\dfrac{-6 \pm 2\sqrt{6}}{6}} = \dfrac{2(-3 \pm \sqrt{6})}{2(3)} = \dfrac{-3 \pm \sqrt{6}}{3}$

WebAssign Stopped Here?

$\dfrac{(-3+\sqrt{6})}{3}, \dfrac{(-3-\sqrt{6})}{3}$

MOAR 2.3 !

19. 0/2 points

Find the first and second derivatives of the function.

$$f(x) = \frac{x^2}{9+4x}$$

$f'(x) =$ $\frac{4x^2+18x}{(9+4x)^2}$

$f''(x) =$ $\frac{162}{(9+4x)^3}$

$$f(x) = \frac{x^2}{9+4x} \qquad \left(\frac{g}{h}\right)' = \frac{g'h - gh'}{h^2}$$

$g = x^2 \qquad h = 4x+9$

$g' = 2x \qquad h' = 4$

$$\frac{g'h - gh'}{h^2} = \frac{(2x)(4x+9) - (x^2)(4)}{(4x+9)^2} =$$

$$= \frac{8x^2 + 18x - 4x^2}{(4x+9)^2} = \boxed{\frac{4x^2+18x}{(4x+9)^2} = f'(x)}$$

§2.5 Chain Rule Makes this Quicker!

$f''(x)$:

$g = 4x^2 + 18x \qquad \boxed{h = (4x+9)^2 = (4x)^2 + 2(4x)(9) + (9)^2}$

$= 16x^2 + 72x + 81$

$g' = 8x + 18 \qquad h' = 32x + 72 \qquad 8(4x+9)$

$$\frac{g'h - gh'}{h^2} = \frac{(8x+18)(4x+9)^2 - (4x^2+18x)(32x+72)}{((4x+9)^2)^2}$$

$$= \frac{(4x+9)\left[(8x+18)(4x+9)' - (4x^2+18x)(8)\right]}{(4x+9)^4} \to 3$$

$$\begin{array}{r} 18 \\ 8 \\ \hline 144 \end{array}$$

$$= \frac{32x^2 + 72x + 72x + 162 - (32x^2 + 144x)}{(4x+9)^3}$$

$$= \frac{32x^2 + 144x + 162 - 32x^2 - 144x}{(\quad)^3} = \boxed{\frac{162}{(4x+9)^3} = f''(x)}$$

$h(x) = (4x+9)^2 = f(g(x))$, where $f(x) = x^2$

$f(\square) = \square^2$

$g(x) = 4x+9$

$h'(x) = 2(4x+9)^1 \cdot (4) = (8x+18)(4) = 32x+72$

$= \frac{df}{dg} \cdot \frac{dg}{dx}$

Way quicker than expanding it out.