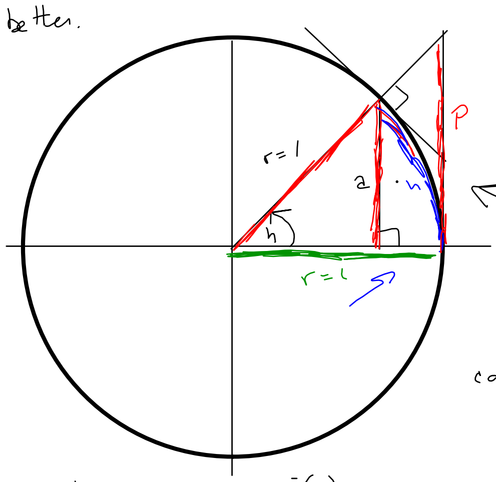
Today: §2.4 $\frac{d}{dx}[\sin(x)] = \cos(x)$

I have a video up on it. I think we can do it a little better.



$\frac{\sin(h)}{h} \xrightarrow{h\to 0} 1$

$\frac{\cos(h)-1}{h} \xrightarrow{h\to 0} 0$

Technique will be to SQUEEZE $\frac{\sin h}{h}$ between $\cos(h)$ & 1, i.e., if we can get $\cos(h) < \frac{\sin(h)}{h} < 1$

then

$\cos(h) \xrightarrow{h\to 0} \cos(0) = 1$

& $1 = 1$

$\lim_{h\to 0} \frac{\sin(h)}{h} = 1$

1st Goal: Show $\frac{\sin(h)}{h} < 1$

Trick: observe $\sin(h) < h$

$\Rightarrow \frac{\sin(h)}{h} < 1$

use picture!

$\sin(h) = \frac{a}{r} = \frac{a}{1} = a$

$\sin(h) = a < h$

$\Rightarrow \frac{\sin(h)}{h} < 1$

Also, Arclength for radians $s = r\theta = \theta = h$!

2nd goal: Show $\cos(h) < \frac{\sin(h)}{h}$.

$\Leftrightarrow h\cos(h) < \sin(h)$

$\Leftrightarrow h < \frac{\sin(h)}{\cos(h)} = \tan(h)$

By picture $h < P = \frac{P}{1} = \tan(h) = \frac{\sin(h)}{\cos(h)}$

$h < \frac{\sin(h)}{\cos(h)} \Rightarrow$

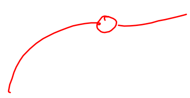
$\cos(h) < \frac{\sin(h)}{h}$

combine w/ $\frac{\sin(h)}{h} < 1$

$\Rightarrow \cos(h) < \frac{\sin(h)}{h} < 1$

$h \to 0$ (each term)

$1 \leq \frac{\sin(h)}{h} \leq 1$



$\therefore \frac{\sin(h)}{h} \xrightarrow{h\to 0} 1$

That's half of it. The other half is to show

$\frac{\cos(h)-1}{h} \xrightarrow{h\to 0} 0$

$$\overset{(a-b)}{\left(\frac{\cos(h)-1}{h}\right)}\overset{(a+b)}{\left(\frac{(\cos(h)+1)}{(\cos(h)+1)}\right)} = \frac{\overset{a^2-b^2}{\cos^2(h)-1}}{h(\cos(h)+1)} = \frac{1-\sin^2(h)-1}{h(\cos(h)+1)}$$

$$= \frac{-\sin^2(h)}{h(\cos(h)+1)} = \left(\frac{\sin(h)}{h}\right)\left(-\frac{\sin(h)}{\cos(h)+1}\right)$$

$\frac{\sin(h)}{h} \xrightarrow{h\to 0} 1$, $\frac{\sin(h)}{\cos(h)+1} \xrightarrow{h\to 0} \frac{1}{2}$

Recall $\lim(fg) = (\lim f)(\lim g)$, provided $\lim f$ & $\lim g$ both exist. & both

$\lim\limits_{h\to 0} \frac{\sin h}{h}$ & $\lim\limits_{h\to 0} \frac{-\sin(h)}{\cos(h)+1} = \frac{0}{2}$ exist.

$$\therefore \left(\frac{\sin(h)}{h}\right)\left(-\frac{\sin(h)}{\cos(h)+1}\right) \xrightarrow{h\to 0} (1)\left(-\frac{0}{2}\right) = 0$$

$$\Rightarrow = (\sin(x))\underbrace{\left(\frac{\cos(h)-1}{h}\right)}_{\to 0} + (\cos(x))\left(\frac{\sin(h)}{h}\right) \xrightarrow{h\to 0}$$

$$\sin(x)(0) + \cos(x)(1) = \cos(x)$$

$$\boxed{\frac{d}{dx}[\sin(x)] = \cos(x)}$$

Claim:

$$\boxed{\frac{d\cos(x)}{dx} = -\sin(x)}$$

$$\frac{\cos(x+h) - \cos(x)}{h} = \frac{\cos(x)\cos(h) - \sin(x)\sin(h) - \cos(x)}{h}$$

$$= \frac{\cos(x)(\cos(h) - 1)}{h} - \frac{\sin(x)\sin(h)}{h}$$

$$= (\cos(x))\left(\frac{\cos(h)-1}{h}\right) - (\sin(x))\left(\frac{\sin(h)}{h}\right) \xrightarrow{h \to 0} -\sin(x)$$

$\frac{\cos(h)-1}{h} \to 0$, $\frac{\sin(h)}{h} \to 1$

Now for the rest of 'em. Quotient Rule $\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$

$$\frac{d}{dx}\left[\tan(x)\right] = \frac{d}{dx}\left[\frac{\sin(x)}{\cos(x)}\right]$$

$$= \frac{\cos(x)\cos(x) - \sin(x)(-\sin(x))}{\cos^2(x)} = \frac{\cos^2(x) + \sin^2(x)}{\cos^2(x)} = \frac{1}{\cos^2(x)}$$

$$= \boxed{\sec^2(x) = \frac{d}{dx}\left[\tan(x)\right]}$$

$$\frac{d}{dx}\left[\sec(x)\right] = \frac{d}{dx}\left[\frac{1}{\cos(x)}\right] = \frac{d}{dx}\left[\frac{f}{g}\right] = \left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$$

$f = 1$, $g = \cos(x)$

$$= \frac{(0)(\cos(x)) - (1)(-\sin(x))}{\cos^2(x)} = \frac{\sin(x)}{\cos^2(x)} = \frac{1}{\cos(x)} \cdot \frac{\sin(x)}{\cos(x)}$$

$$= \boxed{\sec(x)\tan(x) = \frac{d}{dx}\left[\sec(x)\right]}$$

You should try:

$$\boxed{\frac{d}{dx}\left[\csc(x)\right] = -\csc(x)\cot(x)}$$

$$\boxed{\frac{d}{dx}\left[\cot(x)\right] = -\csc^2(x)}$$

Don't do $\lim\limits_{h \to 0}$ things.
Just use previous derivatives, and product/quotient rule.

1. 0/1 points

Differentiate.

$f(x) = x^2 \sin(x)$

$f'(x) =$

$x^2 \cos(x) + 2x \sin(x)$

$(fg)' = f'g + fg'$

$f = x^2, \quad g = \sin(x)$

$f' = 2x, \quad g' = \cos(x)$

$\boxed{2x \sin(x) + x^2 \cos(x)}$

2. 0/1 points

$f'g + fg'$

Differentiate.

$y = \sec(\theta)\tan(\theta)$

$\underbrace{(\sec(\theta)\tan\theta)}^{f'}\overbrace{\tan\theta}^{g} + \overbrace{(\sec\theta)}^{f}\overbrace{(\sec^2(\theta))}^{g'}$

Perfect for hand-written test.

$\sec(\theta)\tan^2(\theta) + \sec^3(\theta)$

$= \sec(\theta)\left[\tan^2\theta + \sec^2\theta\right]$

$= \sec(\theta)\left[\tan^2(\theta) + \tan^2(\theta) + 1\right]$

$= \sec(\theta)\left[2\tan^2(\theta) + 1\right]$

3. 0/1 points

S2.4 #3

Differentiate.

$$y = \frac{2x}{9 - \cot(x)}$$

$$\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$$

Nope

$$\Rightarrow y' = \frac{dy}{dx} = \frac{d}{dx}[y] = \frac{(2)(9-\cot(x)) - 2x(-\csc^2(x))}{(9-\cot(x))^2}$$

Test answer for Mills.

Teacher's in hurry, trying not to waste people's time.

$$\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$$

$g f' - f g'$

Book

I missed $g(x) = 9 - \cot(x)$

$g'(x) = - (-\csc^2(x) = \csc^2(x)$

Ask me about this stuff!
I have fun lecturing.
I want lecture to be more about your questions & especially questions that go deeper, like cool application questions.

$(fgh)' = ???$

4. 0/1 points

Differentiate.

$$f(\theta) = \theta \cos(\theta) \sin(\theta)$$

$(fgh)' = f'gh + fg'h + fgh'$

STOP ON HAND WRITTEN TEST

$= 1(\cos\theta \sin\theta) + \theta(-\sin(\theta))\sin(\theta) + \theta(\cos(\theta))\cos\theta$

$= \cos\theta \sin\theta - \theta \sin^2\theta + \theta\cos^2\theta$

$\cos^2\theta - \sin^2\theta = 1 - \sin^2\theta - \sin^2\theta = 1 - 2\sin^2\theta$

$= \cos(2\theta)$, if I remember my trig identities, correctly.

§2.5 Preview (cnt'd) Chain Rule.

$$\frac{d}{dx}\left[f(g(x))\right] = \frac{d\left(f(g(x))\right)}{dx} = \frac{d f(g(x))}{d g(x)} \cdot \frac{d g(x)}{dx}$$

Think of it as a single variable

Outside-in

$$\frac{d}{dx}\left[f(\boxed{})\right] = \frac{df}{d\boxed{}} \cdot \frac{d\boxed{}}{dx}$$

$$\frac{d}{dx}\left[\sin(x^3-5x)\right] = \frac{d \sin(x^3-5x)}{d(x^3-5x)} \cdot \frac{d(x^3-5x)}{dx}$$

$$= \cos(x^3-5x)\cdot(3x^2-5)$$

$$= (3x^2-5)\cos(x^3-5x)$$ for style points, which don't actually count.

With the chain rule, we can do a new proof of $\frac{d}{dx}[\sec(x)] = \sec(x)\tan(x)$

$$\frac{d}{dx}[\sec(x)] = \frac{d}{dx}\left[\frac{1}{\cos(x)}\right] = \frac{d}{dx}\left[(\cos(x))^{-1}\right]$$

$$= -1\cos(x)^{-2}\cdot(-\sin(x)) = \frac{\sin(x)}{\cos^2(x)} = \frac{\sin(x)}{\cos(x)}\cdot\frac{1}{\cos(x)}$$

$$= \tan(x)\sec(x) = \sec(x)\tan(x)$$

That's the Chain Rule helping.

Special case: Generalized Power Rule:

$$\frac{d}{dx}\left[(f(x))^n\right] = n\, f(x)^{n-1} f'(x)$$

$$= n(f(x))^{n-1}\frac{df}{dx}$$