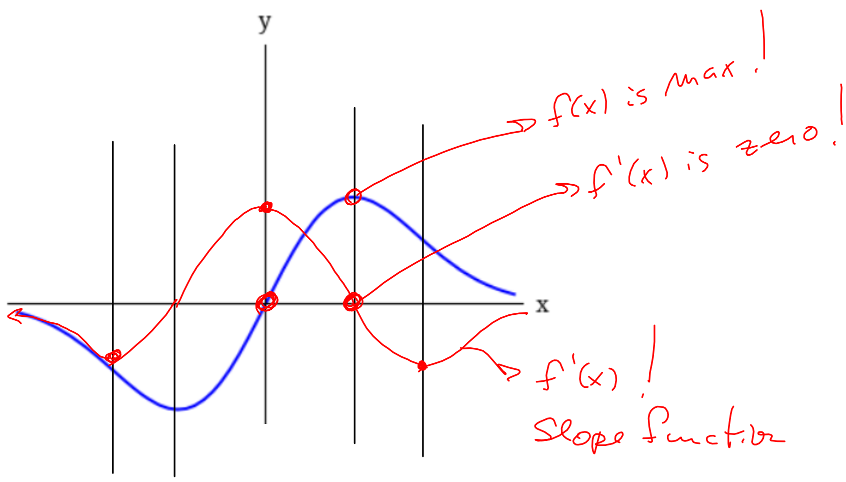
1. 0/1 points

Use the given graph of $f(x)$ to sketch the graph of f'.



#7 on 2.2:

Find $f'(x)$ by the definition.

$f(x) = \sqrt{5-x}$

$$\frac{f(x+h)-f(x)}{h} = \left(\frac{\sqrt{5-(x+h)} - \sqrt{5-x}}{h}\right)\left(\frac{\sqrt{5-(x+h)} + \sqrt{5-x}}{\sqrt{5-(x+h)} + \sqrt{5-x}}\right)$$

$$= \frac{5-(x+h)-(5-x)}{h\left(\sqrt{5-(x+h)}+\sqrt{5-x}\right)} = \frac{5-x-h-5+x}{\sim} = \frac{-\cancel{h}}{\cancel{h}\left(\sqrt{5-(x+h)}+\sqrt{5-x}\right)}$$

$$= \frac{-1}{\sqrt{5-(x+\underbrace{h}_{\to 0})}+\sqrt{5-x}} \xrightarrow{h\to 0} \frac{-1}{\sqrt{5-x}+\sqrt{5-x}} = \frac{-1}{2\sqrt{5-x}}$$

Cheat: We don't have the chain rule yet!

Chain Rule:

$$\frac{d}{dx}\left[f(x)\right] = f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

Liebniz Notation

The derivative, with respect to x, of $f(x) = f'(x)$

$$\frac{d}{dx}\left[g(h(x))\right] = \frac{dg}{dh}\cdot\frac{dh}{dx}$$

How fast what's being fed to it changes with respect to x.

How fast g changes with respect to what's being fed to it.

$$\frac{d}{dx}\left[(x^2+3x)^5\right]$$

$f(x) = g(h(x))$, where

$g(x) = x^5 \implies g'(x) = 5x^4$

$h(x) = x^2+3x \implies h'(x) = 2x+3$

$$\frac{dg}{dh}\cdot\frac{dh}{dx} = g'(h(x))\cdot h'(x)$$

$g'(h(x))$

$= 5h(x)^4 = 5(x^2+3x)^5$

So $f'(x) = 5(x^2+3x)^4(2x+3)$

$\sqrt{5-x} = f(x)$

$g(x) = \sqrt{x} = x^{\frac{1}{2}} \implies g'(x) = \frac{1}{2}x^{-\frac{1}{2}} \implies g'(h(x)) = \frac{1}{2}h(x)^{-\frac{1}{2}}$

$= \frac{1}{2}(5-x)^{-\frac{1}{2}}$

$h(x) = 5-x \implies h'(x) = -1$

$$g'(h(x))\,h'(x) = \frac{1}{2}(5-x)^{-\frac{1}{2}}(-1) = \frac{-1}{2(5-x)^{\frac{1}{2}}} = \boxed{\frac{-1}{2\sqrt{5-x}}}$$

Same as we got the hard way!

One more chain rule to keep up with students forging ahead!

$f(x) = \sqrt[4]{x^3 - 57x} = (x^3 - 57x)^{\frac{1}{4}}$, with domain $\mathcal{D} = \{x \mid x^3 - 57x \geq 0\}$

$\Rightarrow f'(x) = \frac{1}{4}(x^3 - 57x)^{\frac{1}{4}-1}(3x^2 - 57)$

$= \frac{1}{4}(x^3 - 57x)^{-3/4}(3x^2 - 57)$

$= \dfrac{3x^2 - 57}{4\sqrt[4]{(x^3 - 57x)^3}}$ OR $\dfrac{3x^2 - 57}{4\sqrt[4]{x^3 - 57x}^{\,3}}$

Why didn't anybody respond to my "Does this make sense?" question?

§2.4

$$\frac{d}{dx}\left[\sin(x)\right] = \cos(x)$$

$$\frac{d}{dx}\left[\cos(x)\right] = -\sin(x)$$

$$\frac{d}{dx}\left[\tan(x)\right] = \sec^2(x)$$

Recall Product Rule: $(fg)' = f'g + fg'$

$$\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$$

$$\frac{d}{dx}\left[\tan(x)\right] = \frac{d}{dx}\left[\frac{\sin(x)}{\cos(x)}\right] = \frac{\overset{f'}{(\cos(x))}\overset{g}{(\cos(x))} \overset{-}{-} \overset{f}{\sin(x)}\overset{g'}{(-\sin(x))}}{(\cos(x))^2}$$

$$= \frac{\cos^2(x) + \sin^2(x)}{\cos^2(x)} = \frac{1}{\cos^2(x)} = \sec^2(x)$$

$$\frac{\cos^2 x}{\cos^2 x} + \frac{\sin^2 x}{\cos^2 x} = \boxed{1 + \tan^2(x) = \sec^2(x)}$$

An old identity from trig!

9. 0/1 points

Differentiate.

$$F(y) = \left(\frac{1}{y^2} - \frac{3}{y^4}\right)(y + 5y^3) = (y^{-2} - 3y^{-4})(y + 5y^3)$$

$$5 + \frac{14}{y^2} + \frac{9}{y^4}$$

$\Rightarrow F'(x) = f'g + fg'$

FORGOT TO EXPAND THIS,

$= (-2y^{-3} - (-12)y^{-5})(y + 5y^3) + \boxed{(y^{-2} - 3y^{-4})(1 + 15y^2)}$

Scratch

$(-2y^{-3} + 12y^{-5})(y + 5y^3) = -2y^{-2} - 10y^{0} + 12y^{-4} + 60y^{-2}$

$-3+1$, $-3+3$, $-5+1$, $-5+3$

$= 58y^{-2} - 10 + 12y^{-4}$

$= -10 + 58y^{-2} + 12y^{-4}$

Looks wrong!

Different Approach

$= (y^{-2} - 3y^{-4})(y + 5y^3) = y^{-2+1} + 5y^{-2+3} - 3y^{-4+1} - 15y^{-4+3}$

$= y^{-1} + 5y - 3y^{-3} - 15y^{-1}$

$= -14y^{-1} + 5y - 3y^{-3}$ Looks easier!

$= 14y^{-2} + 5 + 9y^{-4}$

$= \boxed{\frac{14}{y^2} + 5 + \frac{9}{y^4}}$ is good.

So product Rule was harder this time.

Find $f'(x)$ for $f(x) = (x^2 - 5x)^{15}(3x^{-1} + 2)^{11}$

Hand-written test: final answer:

$f'(x) = 15(x^2-5x)^{14}(2x-5)(3x^{-1}+2)^{11} + (x^2-5x)^{15}\left(11(3x^{-1}+2)^{10}\right)(-3x^{-2})$

Nobody wants to expand $(\sim)^{15}, (\sim)^{11}, (\sim)^{14}$

§ 2.3 Tangent Line Question, only now the cheats are not cheating, any more.

$\frac{4x}{x+2}$ @ $(2,2)$

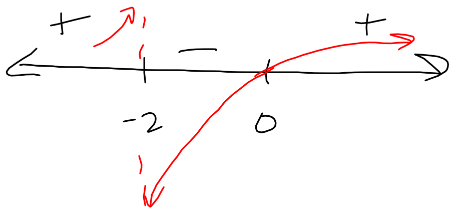
Review:

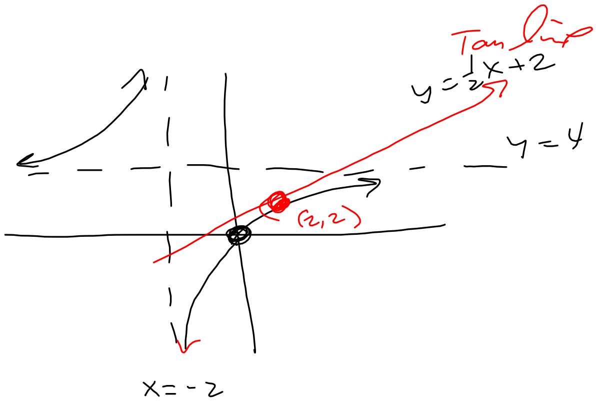
Plot $\frac{4x}{x+2}$

$\boxed{\text{V.A.}: \; x=-2}$

H.A.: $\frac{4x}{x} \longmapsto \boxed{4=y}$

x-int: $(0,0)$





work the problem:

$$r(x)=\frac{4x}{x+2} \Rightarrow r'(x)=\frac{4(x+2)-4x(1)}{(x+2)^2}=\frac{4x+8-4x}{(x+2)^2}=\frac{8}{(x+2)^2}$$

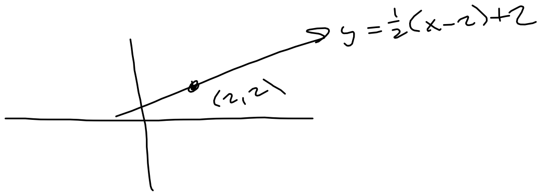
want $r'(2)=\frac{8}{(2+2)^2}=\frac{8}{16}=\frac{1}{2}=m_{tan}=r'(2)$, $(x_1,y_1)=(2,2)$

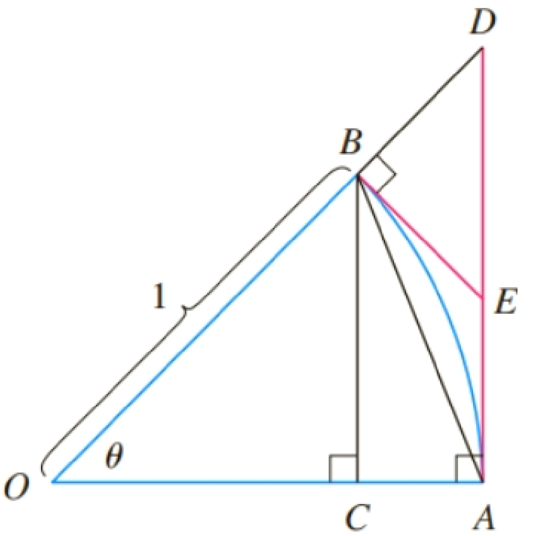
$$y=m(x-x_1)+y_1=r'(2)(x-2)+r(2)$$

$$\boxed{=\frac{1}{2}(x-2)+2}$$

$$=\frac{1}{2}x-1+2=\frac{1}{2}x+1$$

$L(x)=\frac{1}{2}x+1$ for WebAssign.





(a)

Have this figure ready to go for Wednesday!

We'll be using this to prove the derivative of sine is cosine, either today, or Wednesday, but the plan is Wednesday.

Claim:

$$\frac{d}{dx}\left[\sin(x)\right] = \cos(x)$$

Proof:

$$\frac{\sin(x+h) - \sin(x)}{h}$$

$$= \frac{\sin(x)\cos(h) + \sin(h)\cos(x) - \sin(x)}{h}$$

$$= \frac{(\sin(x))(\cos(h)-1) + (\cos(x))(\sin(h))}{h}$$

$$= (\sin(x))\underbrace{\left(\frac{\cos(h)-1}{h}\right)}_{\to 0} + (\cos(x))\overbrace{\left(\frac{\sin(h)}{h}\right)}^{\to 1}$$

WANT $\xrightarrow{h \to 0} \cos(x)$

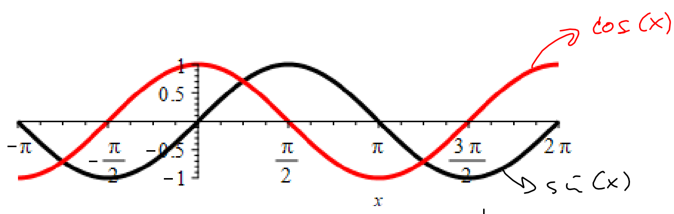
We need to show $\frac{\cos(h)-1}{h} \xrightarrow{h \to 0} 0$

& $\frac{\sin(h)}{h} \xrightarrow{h \to 0} 1$.

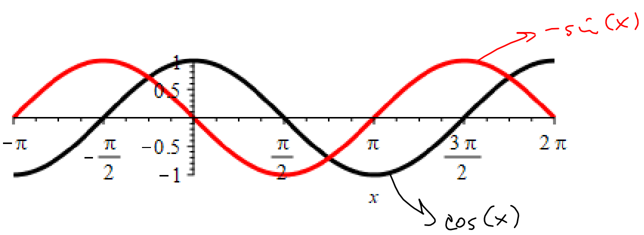
Then $\frac{d}{dx}\left[\sin(x)\right] = \cos(x)$, as desired

We need the picture to do that.

Graphical Appeal to your Intuition: $\frac{d}{dx}\left[\sin(x)\right] = \cos(x)$



cosine is sine's slope!



22. 0/2 points

Find constants A and B such that the function $y = A\sin(x) + B\cos(x)$ satisfies the differential equation $y'' + y' - 3y = \sin(x)$.

$y' = A\cos(x) - B\sin(x)$

$y'' = -A\sin(x) - B\cos(x)$ $\Rightarrow y'' + y' - 3y$

$= -A\sin(x) - B\cos(x) + A\cos(x) - B\sin(x) - 3(A\sin(x) + B\cos(x))$

$= -A\sin(x) - B\cos(x) + A\cos(x) - B\sin(x) - 3A\sin(x) - 3B\cos(x) \overset{\text{WANT}}{=} \sin(x)$

$\Rightarrow -A\sin(x) - B\sin(x) - 3A\sin(x) = \sin(x)$

$-4A\sin(x) - B\sin(x) = \sin(x) \Rightarrow$

$\boxed{-4A - B = 1}$

$-B\cos(x) + A\cos(x) - 3B\cos(x) = 0$

$-4B\cos(x) + A\cos(x) = 0$

$\cos(x)(-4B + A) = 0 \Rightarrow$

$\cos(x) = 0$ everywhere (Nope)

or $\boxed{-4B + A = 0}$

$$\left.\begin{aligned} -4A - B &= 1 \\ A - 4B &= 0 \end{aligned}\right\}$$

Substitution:

$A - 4B = 0$

$A = 4B$

$-4(4B) - B = 1$

$-16B - B = 1$

$-17B = 1$

$\boxed{B = -\frac{1}{17}}$

$A = 4B = \boxed{-\frac{4}{17} = A}$

$A - 4B = 0$

$-4A - B = 1$

$$\left[\begin{array}{cc|c} 1 & -4 & 0 \\ -4 & -1 & 1 \end{array}\right]$$

$$\sim \left[\begin{array}{cc|c} 1 & -4 & 0 \\ 0 & -17 & 1 \end{array}\right]$$

~~Matrix~~ Gaussian Elimination w/ matrix

$-17B = 1$

$\boxed{B = -\frac{1}{17}}$

$A - 4B = 0 \Rightarrow$

$A - 4\left(-\frac{1}{17}\right) = 0$

$A + \frac{4}{17} = 0$

$\boxed{A = -\frac{4}{17}}$