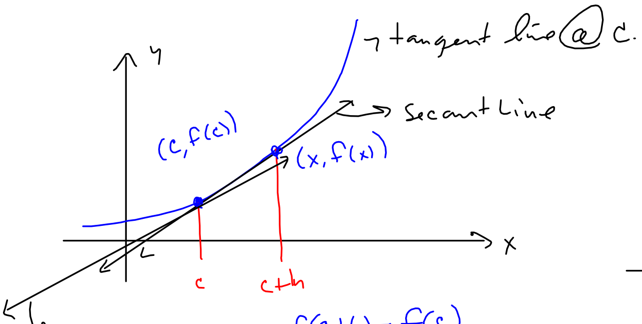
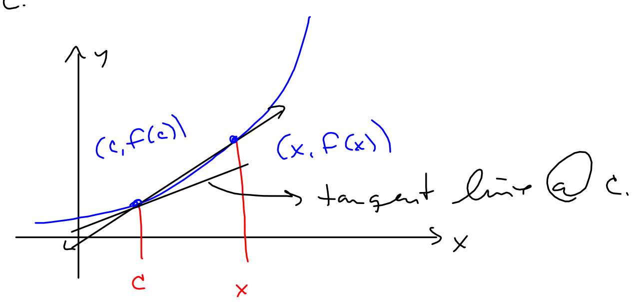
Finishing up Sections 2.1, 2.2



tangent line @ c

$$f'(c) = \lim_{h \to 0} \frac{f(c+h) - f(c)}{h}$$



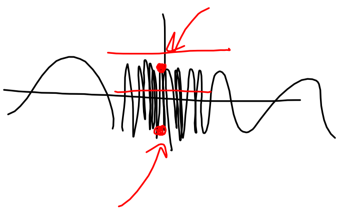
$$f'(c) = \lim_{x \to c} \frac{f(x) - f(c)}{h}$$

$$f(x) = \begin{cases} x \sin\left(\frac{\pi}{x}\right) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

. Is it difbl @ $x=0$?

$$f'(0) = \lim_{h\to 0} \frac{f(0+h) - f(0)}{h} = \lim_{h\to 0} \frac{f(h) - f(0)}{h} = \lim_{h\to 0} \frac{h \sin\left(\frac{\pi}{h}\right) - 0}{h}$$

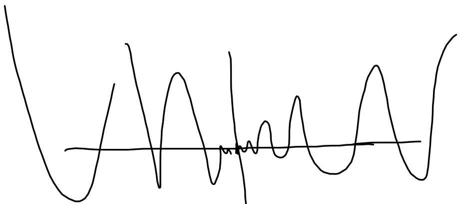
$$= \lim_{h\to 0} \frac{h\left(\sin\left(\frac{\pi}{h}\right)\right)}{h} = \lim_{h\to 0} \sin\left(\frac{\pi}{h}\right) \nexists$$



No

Does $f'(0)$ exist for

$$f(x) = \begin{cases} x^2 \sin\left(\frac{\pi}{x}\right) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$



$$\lim_{h\to 0} \frac{f(0+h) - f(0)}{h} = \lim_{h\to 0} \frac{h^2 \sin\left(\frac{\pi}{h}\right) - 0}{h} = \lim_{h\to 0} h \sin\left(\frac{\pi}{h}\right) = 0 \; !$$

$$f'(0) = 0 \; !$$

Recall the cheat: $\frac{d}{dx}\left[x^n\right] = n x^{n-1} \quad \forall n \neq 0$

$f(x) = x^n \Rightarrow f'(x) = n x^{n-1} \quad \forall n \neq 0$

$\frac{d}{dx}\left[x^0\right] = \frac{d}{dx}\left[1\right] = 0$ Slope of a horizontal line.

$\frac{d}{dx}\left[3x^5\right] = 3\frac{d}{dx}\left[x^5\right] = 3\left(5x^4\right] = 15x^4$

§2.2 #16 $x^2 - \sqrt{x} + 1 = f(x)$

cheat; $= x^2 - x^{\frac{1}{2}} + 1 \Rightarrow \boxed{f'(x) = 2x - \frac{1}{2}x^{-\frac{1}{2}}}$

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{(x+h)^2 - \sqrt{x+h} + 1 - (x^2 - \sqrt{x} + 1)}{h}$$

$$= \lim_{h\to 0}\frac{(x+h)^2 - x^2}{h} - \lim_{h\to 0}\frac{\sqrt{x+h} - \sqrt{x}}{h} + \lim_{h\to 0}\frac{1-1}{h} = A + B + C$$

$$= \underset{A}{2x} - \underset{+B}{\frac{1}{2\sqrt{x}}} \underset{+C}{+0} = 2x - \frac{1}{2\sqrt{x}} = 2x - \frac{1}{2}x^{-\frac{1}{2}}$$

$\sqrt{x^2} = |x|$

$(\sqrt{x})^2 = x$

$\frac{1-1}{h} = 0$

$\lim_{h\to 0}\frac{1-1}{h} = \lim_{h\to 0} 0 = 0$

$g(x) = 1$ is constant function

$g(x+h) = 1$

17. Question Details SCalc8 2.1.053. [3354491]

The cost of producing x ounces of gold from a new gold mine is $C = f(x)$ dollars.

(a) What is the meaning of the derivative $f'(x)$? What are its units?

(b) What does the statement $f'(700) = 17$ mean?

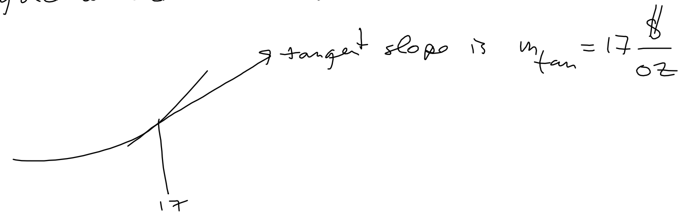
(c) Do you think the values of $f'(x)$ will increase or decrease in the short term? What about the long term? Explain.

Lexicon
x = production of gold, in ounces of gold.
$f = f(x)$ = cost of producing x ounces of gold, in dollars.

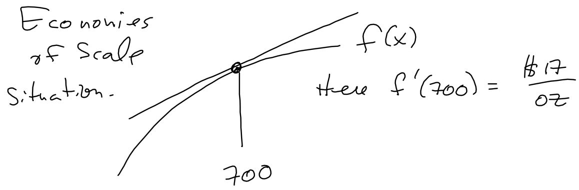
(a) $f'(x)$ is the rate of change in cost of producing x ounces of gold.
"the incremental change in the cost of producing gold with respect to an incremental change in the amount of gold produced."

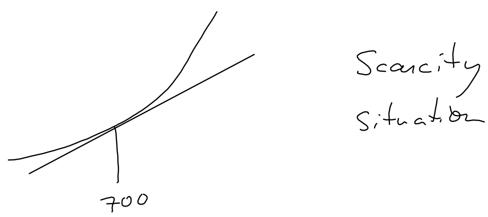
(b) $f'(700) = 17$ means the rate of change in cost with respect to an incremental change in the amount of gold produced is \$ 17/ounce at a production level of 700 ounces.

MARGINAL COST would say "The cost of the 701^{st} ounce of gold would be \$17."

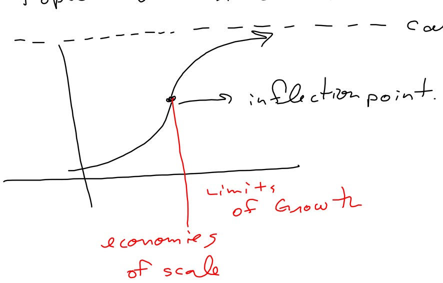


(c) Do you think $f'(x)$ will increase or decrease in the short term?
I think this may be poorly posed!





Population Growth Model: Basic Logistic Model



§2.3 Product Rule, Quotient Rule

If f & g are differentiable (diffble) and $r(x) = (fg)(x)$
$= f(x)\,g(x)$, then

$r'(x) = f'(x)g(x) + f(x)g'(x)$

Book gives it in a different form for NO GOOD REASON.

$$\boxed{(fg)' = f'g + fg'}$$

Quotient Rule:

$s(x) = \dfrac{f(x)}{g(x)}$, Then $s'(x) = \dfrac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}$

$$\boxed{\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}}$$

$$\boxed{\begin{array}{l}(f \pm g)' = f' \pm g' \\ (3f)' = 3f'\end{array}}$$

Derivative is a linear operator, by props of limits.

Product Rule Proof: $r'(x)$

$$r'(x) = \lim_{h\to 0} \frac{r(x+h) - r(x)}{h} = \lim_{h\to 0} \frac{f(x+h)g(x+h) - f(x)g(x)}{h}$$

$$= \lim_{h\to 0} \frac{f(x+h)g(x+h) - f(x)g(x+h) + f(x)g(x+h) - f(x)g(x)}{h}$$

$$= \lim_{h\to 0} \frac{f(x+h)g(x+h) - f(x)g(x+h)}{h} + \lim_{h\to 0} \frac{f(x)g(x+h) - f(x)g(x)}{h} \Big)$$

$$= \lim_{h\to 0} \frac{(f(x+h) - f(x))g(x+h)}{h} + \lim_{h\to 0} \frac{f(x)(g(x+h) - g(x))}{h}$$

$$= \lim_{h\to 0} \left[\left(\frac{f(x+h) - f(x)}{h}\right) g(x+h)\right] + \lim_{h\to 0} f(x) \left[\frac{g(x+h) - g(x)}{h}\right]$$

Now, f & g are difb$^{\underline{l}}$ which implies they're cnt$^{\underline{S}}$

$$= \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} \lim_{h\to 0} g(x+h) + f(x) \lim_{h\to 0} \frac{g(x+h) - g(x)}{h}$$

$$= f'(x)\, g(x) + f(x)\, g'(x) \quad \blacksquare$$

Quotient Rule (Cheating §2.2 #8)

$$r(t) = \frac{1-3t}{5+t} \Rightarrow r(t) = \frac{f(t)}{g(t)}$$

$$\Rightarrow r'(t) = \frac{f'(t)g(t) - f(t)g'(t)}{g(t)^2} = \frac{f'g - fg'}{g^2}$$

$f(t) = 1-3t^1 \Rightarrow f'(t) = -3 \cdot 1 \cdot t^0 = -3$

$g(t) = 5+t \Rightarrow g'(t) = 1$

$$\Rightarrow r'(t) = \frac{f'g - fg'}{g^2} = \frac{(-3)(5+t) - (1-3t)(1)}{(5+t)^2}$$

$$= \frac{-15-3t-1+3t}{(t+5)^2} = \frac{-16}{(t+5)^2}$$

Limit Definition Method Owie!

$$\frac{r(t+h) - r(t)}{h} = \frac{\frac{1-3(t+h)}{5+(t+h)} - \frac{1-3t}{5+t}}{h}$$

$$= \frac{1}{h}\left[\frac{(1-3(t+h))(5+t) - (1-3t)(5+t+h)}{(5+t+h)(5+t)}\right] \quad LCD = (5+t+h)(5+t)$$

$$= \frac{1}{h}\left[\frac{(1-3t-3h)(5+t) - [5+t+h-15t-3t^2-3ht]}{LCD}\right]$$

$$= \frac{1}{h}\left[\frac{5+t-15t-3t^2-15h-3ht-5-t-h+15t+3t^2+3ht}{LCD}\right]$$

$$= \frac{1}{\cancel{h}}\left[\frac{-16\cancel{h}}{(5+t+h)(5+t)}\right] \xrightarrow{h \to 0} \boxed{\frac{-16}{(5+t)^2} = r'(t)}$$

$h \to 0$

$\frac{gf' - fg'}{g^2}$ Book? Ewwwww