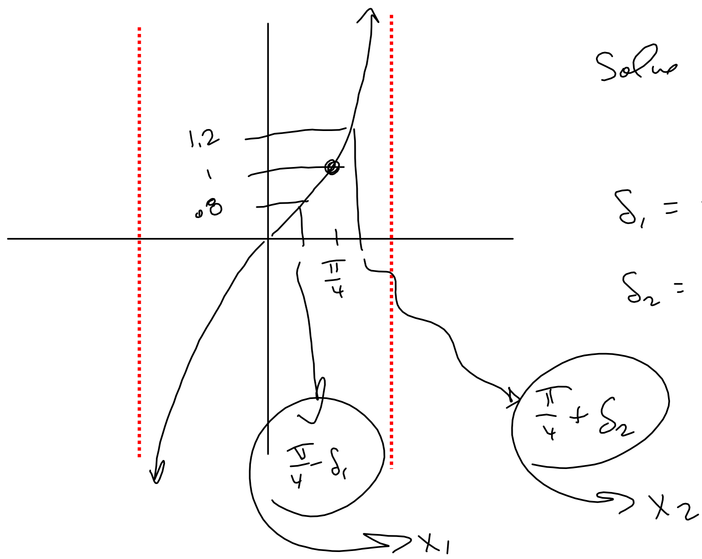


Solve $\tan(x_2) = 1.2$

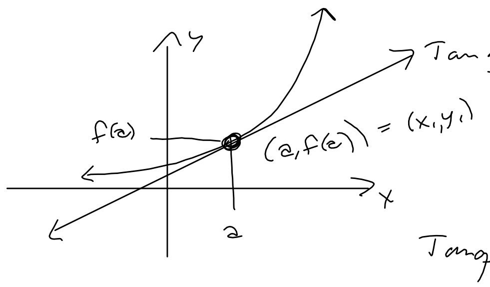
$\tan(x_1) = .8 \Rightarrow x_1 = \arctan(.8)$

$\delta_1 = x_2 - \frac{\pi}{4}$

$\delta_2 = \frac{\pi}{4} - x_1$

§2.1 Tangent & Tangent Line.

Define $m_{tan} = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h} = f'(a)$



$y_1 = f(a)$
$x_1 = a$

Tangent Line is, therefore,

$y = m(x - x_1) + y_1$

$\boxed{y = f'(a)(x-a) + f(a)}$

we'll do a million of these!
In 2.1, we'll do it in the most horribly, atrociously difficult way possible. The dreaded

"Definition of the Derivative" way

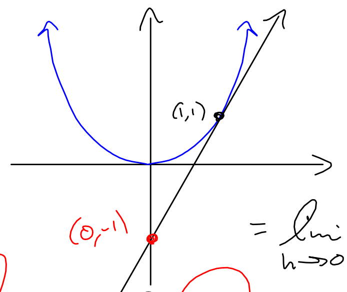
§2.2, we'll give you shortcuts.

Go To harryzaims.com and look at the NOTES.

Try watching a video.

Find the tangent line to $f(x) = x^2$ at $x = 1 = a$

$f(a) = f(1) = 1^2 = 1 \rightsquigarrow (a, f(a)) = (1,1)$



$y = L_a(x) =$ Linearization of $f(x)$ @ $x = 1$

we need $m_{tan} = f'(a) = f'(1)$

$= \lim_{h\to 0} \frac{f(1+h)-f(1)}{h} = \lim_{h\to 0} \frac{(1+h)^2 - 1^2}{h}$

$= \lim_{h\to 0} \frac{1^2 + 2(1)h + h^2 - 1}{h} = \lim_{h\to 0} \frac{2h+h^2}{h} = \lim_{h\to 0} \frac{h(2+h)}{h} = \lim_{h\to 0}(2+h)$

$= 2 = m_{tan} = f'(1)$

$L_1(x) = f'(1)(x-1) + f(1)$

$= 2(x-1) + 1$ is done, AFAIC, but WebAssign's stupid & requires $y = mx+b$

$y = 2x - 2 + 1 = \boxed{2x - 1 = L(x)}$

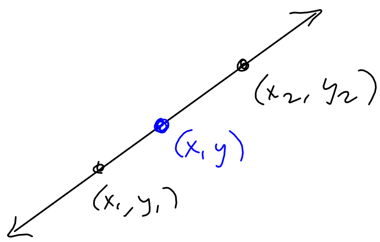
Notation (Shorter): pass to limit after all the work is done:

$$\frac{f(1+h)-f(1)}{h} = \frac{(1+h)^2 - 1^2}{h} = \frac{1+2h+h^2-1}{h}$$

$$= \frac{2h+h^2}{h} = \frac{h(2+h)}{h} \underset{(h \neq 0)}{=} 2+h \xrightarrow{h \to 0} 2 = f'(1)$$

Background:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$



$\forall\ (x_1, y_1),\ (x_2, y_2)$ on $\mathcal{L}$

Let $(x, y) \in \mathcal{L}$

$$\frac{y - y_1}{x - x_1} = m$$

$\Rightarrow y - y_1 = m(x - x_1)$ Book form for Point-Slope.

$y = m(x - x_1) + y_1$ my way & your way for the duration of Calculus I, II, III & beyond.

Cheats;

$\frac{d}{dx}\left[x^n\right] = n x^{n-1}$ Power rule $\forall\ n \neq 0$.

$\frac{d}{dx}\left[x^0\right] = \frac{d}{dx}\left[1\right] = 0$

After §2.1: $f(x) = x^2$. Find eq'n of tangent line to $f(x)$ @ $x=1, y=1$

$f(x) = x^2 \Rightarrow 2x^1 = 2x = f'(x) \Rightarrow$

$f'(1) = 2(1) = 2 = m_{tan}$.

$y = f'(1)(x-1) + f(1)$

$= 2(x-1) + 1$

$= 2x - 2 + 1$

$= \boxed{L_1(x) = 2x - 1}$

§ 2.1 #3 in Notes/video/WebAssign.

$(1+h)^7 = ?$

$$
\begin{array}{c}
1 \\
1 \quad 1 \\
1 \quad 2 \quad 1 \\
1 \quad 3 \quad 3 \quad 1 \\
1 \quad 4 \quad 6 \quad 4 \quad 1 \\
1 \quad 5 \quad 10 \quad 10 \quad 5 \quad 1 \\
1 \quad 6 \quad 15 \quad 20 \quad 15 \quad 6 \quad 1 \\
1 \quad 7 \quad 21 \quad 35 \quad 35 \quad 21 \quad 7 \quad 1
\end{array}
$$

$(1+h)^7 = 1^7h^0 + 7(1)^6(h)^1 + 21(1)^5(h)^2 + 35(1)^4(h)^3 + 35h^4 + 21h^5 + 7h^6 + h^7$

$$\frac{f(1+h)-f(1)}{h} = \frac{(1+h)-(1+h)^7-(1-1^7)}{h}$$

$$= \frac{1+h-(1+7h+21h^2+\dots+h^7)}{h}$$

$$= \frac{\cancel{1}+h-\cancel{1}-7h-21h^2-\dots-h^7}{h} = \frac{-6h-21h^2-35h^3-\dots-h^7}{h}$$

$$= \frac{\cancel{h}(-6-21h-\dots-h^6)}{\cancel{h}} = -6-21h-\dots-h^6 \xrightarrow{h\to 0} -6$$

$(h \neq 0)$

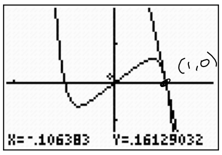
Cheat/check:

$f(x) = x - x^7 \Rightarrow f'(x) = 1-7x^6$

$\Rightarrow f'(1) = 1-7(1)^6 = 1-7 = -6 = m_{tan}$.

$f(1) = 1-1^7 = 0 \qquad (a, f(a)) = (x_1, f(x_1)) = (x_1, y_1) = (1,0)$

$L_1(x) = -6(x-1)+0 = \boxed{-6x+6 = y}$



$y = x - x^7 = x(1-x^6) = -x(x^6-1)$

$= -x(x^6-1) = -x((x^3)^2 - 1^2)$

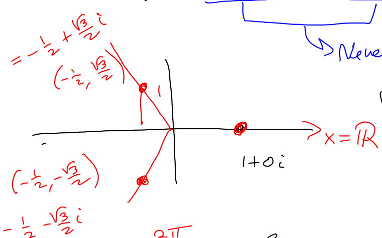
$= -x(x^3-1)(x^3+1)$

$= -x(x-1)\underbrace{(x^2+x+1)(x+1)(x^2-x+1)}$

missed this in my notes.

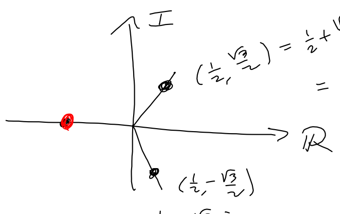
Never have real roots!

Recall DeMoivre's Theorem



$\frac{2\pi}{3}$

$x^3+1=0 \Rightarrow x^3=-1 \Rightarrow x=-1$



$= \cos\frac{\pi}{3} + i\sin\frac{\pi}{3}$

$\frac{\pi}{3} + \frac{2\pi}{3} = \pi$

$\cos\pi + i\sin\pi = -1+0i$

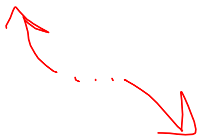
$\pi + \frac{2\pi}{3} = \frac{5\pi}{3}$

$= \frac{1}{2} - \frac{\sqrt{3}}{2}i$

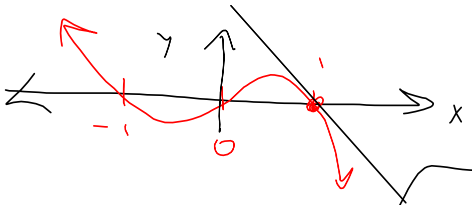
$= \cos(\frac{5\pi}{3}) + i\sin(\frac{5\pi}{3})$

Graph of $x - x^7 = -x(x^6 - 1)$

$-x^7$



$= -x(x-1)(x+1)(x^2+x+1)(x^2-x+1)$



$y = -6x + 6$

WebAssign #5 §2.1:

$f(x) = \frac{7}{\sqrt{x}}$ @ $x=1$, then @ $x=4$. Let's do

$f'(x)$ in general & plug in $x=1$ & $x=4$.

$f(x) = \frac{7}{\sqrt{x}} = 7x^{-\frac{1}{2}} \Longrightarrow$ Cheat says $f'(x) = \left(-\frac{1}{2}\right)(7)x^{-\frac{1}{2}-1}$

$= -\frac{7}{2}x^{-\frac{3}{2}} = -\frac{7}{2\sqrt{x^3}} = -\frac{7}{2\sqrt{x^3}} = f'(x)$

$$\frac{f(x+h)-f(x)}{h} = \frac{\frac{7}{\sqrt{x+h}} - \frac{7}{\sqrt{x}}}{h}$$

$$= \frac{1}{h}\left[\frac{7}{\sqrt{x+h}}\cdot\frac{\sqrt{x}}{\sqrt{x}} - \frac{7}{\sqrt{x}}\cdot\frac{\sqrt{x+h}}{\sqrt{x+h}}\right] = \frac{1}{h}\left[\frac{7\sqrt{x}-7\sqrt{x+h}}{\sqrt{x}\sqrt{x+h}}\right]$$

$$= \frac{1}{h}\left[\frac{7\sqrt{x}-7\sqrt{x+h}}{\sqrt{x}\sqrt{x+h}}\right]\left[\frac{7\sqrt{x}+7\sqrt{x+h}}{7\sqrt{x}+7\sqrt{x+h}}\right]$$

$$= \frac{1}{h}\left[\frac{49x-49(x+h)}{\sqrt{x}\sqrt{x+h}\,(7\sqrt{x}+7\sqrt{x+h})}\right]$$

$$= \frac{1}{h}\left[\frac{49x-49x-49h}{\sqrt{x}\sqrt{x+h}\,(\underline{\qquad})}\right] =$$

$= x$

$$= \frac{1}{\cancel{h}}\left[\frac{-49\cancel{h}}{\underline{\qquad}}\right] = \frac{-49}{\sqrt{x}\sqrt{x+h}\,(7\sqrt{x}+7\sqrt{x+h})}$$

$(h \neq 0)$

$$\xrightarrow{h\to 0} \frac{-49}{\sqrt{x}\sqrt{x}\,(7\sqrt{x}+7\sqrt{x})} = \frac{-49}{x(14\sqrt{x})}$$

$$= \boxed{\frac{-7}{2\underbrace{x\sqrt{x}}}} = \boxed{-\frac{7}{2}x^{-\frac{3}{2}}}$$

$x x^{\frac{1}{2}} = x^{\frac{3}{2}}$

Good chance this is the form WebAssign will want to see.

$\sqrt{x}\,\sqrt{x} = \sqrt{x}^{\,2}$

$\sqrt{x^2} = |x|$

$x^2 = 9 \Rightarrow$

$x = \pm 3$

$x^2 = 9$

$\sqrt{x^2} = \sqrt{9}$

$|x| = 3$

$x = 3$ OR $x = -3$

$x = \pm 3$