12. 0/1 points

Evaluate the limit, if it exists. (If an answer does not exist, enter DNE.).

$$\lim_{h \to 0} \frac{\sqrt{1+h} - 1}{h}$$ $h=0$ gives $\frac{0}{0}$?!

1/2

we did a "rationalize the denominator" example, last time.

Now, we " " " NUMERATOR! "

$$\left(\frac{\sqrt{1+h} - 1}{h}\right)\left(\frac{\sqrt{1+h} + 1}{\sqrt{1+h} + 1}\right) = \frac{1+h-1}{h(\sqrt{1+h}+1)} = \frac{\cancel{h}}{\cancel{h}(\sqrt{1+h}+1)}$$

$$= \frac{1}{\sqrt{h+1}+1} \xrightarrow{h \to 0} \frac{1}{\sqrt{1}+1} = \frac{1}{1+1} = \frac{1}{2} = \lim_{h \to 0} \frac{\sqrt{1+h}-1}{h}$$

This is saying the slope of $f(x) = \sqrt{x}$ is $m_{tan} = f'(1) = \frac{1}{2}$

we've found the slope of the tangent line!

$m = \frac{1}{2}$, passes thru $(x, y) = (x, f(x)) = (1, f(1)) = (1, \sqrt{1})$

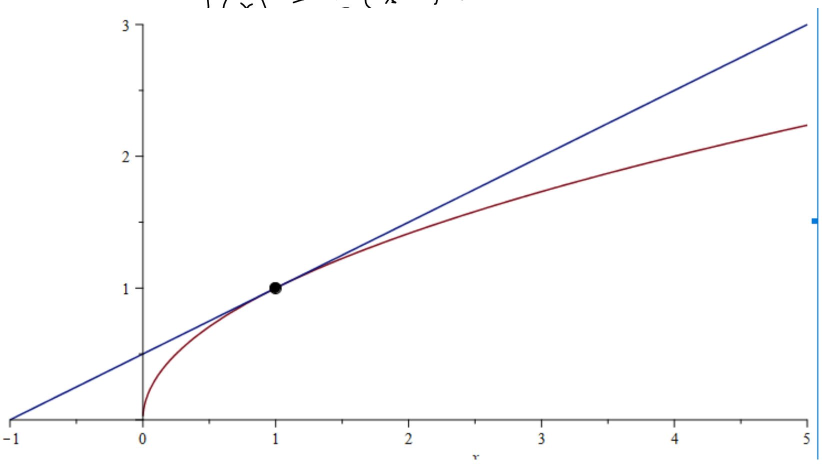
$= (x_1, y_1) = (1, 1) = (1, f(1))$

Tangent line eqn from point-slope:

$$y = m(x - x_1) + y_1$$

$$L(x) = f'(1)(x-1) + f(1)$$

$$L(x) = \frac{1}{2}(x-1) + 1 = \text{Tan Line!}$$



Tangent Line stays close to $\sqrt{x}$ as long as x is "close" to $x = 1$.

"Smooth curves are locally linear."

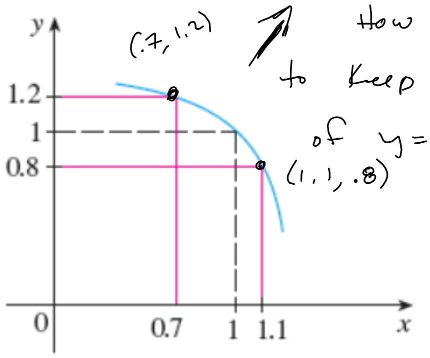
1. 0/1 points SCalc8 1.7.001. [3354476]

Use the given graph of f to find a number δ such that

if $|x - 1| < \delta$ then $|f(x) - 1| < 0.2 = \varepsilon$

$\delta =$ [ ] 0.1

$\lim_{x \to 1} f(x) = 1$



How must we restrict the input to keep the output within 0.2 units of $y = 1$?

Take smaller of 0.3 & 0.1

$\delta = \min\{.3, .1\} = .1 \equiv \delta$

if $|x-1| < .1$ then $|f(x)-1| < .2$

For $|☺| < B$:

$-B < ☺ < B$

$|2x-3| < 5 \implies$

$-5 < 2x-3 < 5 \implies$

$-2 < 2x < 8 \implies$

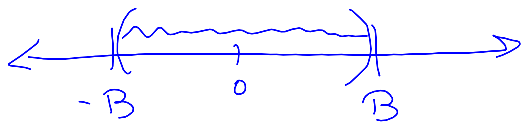
$-1 < x < 4$

More nuts 'n' bolts :

$|☺| < B \implies$

☺ $< B$ AND ☺ $> -B$

☺ lives here:



$|☺| > B$

$-B > ☺ > B$?! No!

$|3x| > 5 \implies$

$-5 > 3x > 5$?!

$-5 > 5$?

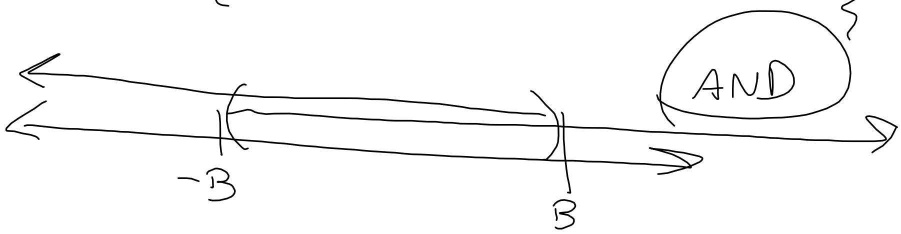
$|☺| > B \implies$

☺ > B OR ☺ < −B

☺ is here, OR THERE

$|A| < B \Rightarrow$ $|A| > B \Rightarrow$

$A < B$ and $A > -B$ $A > B$ OR $A < -B$



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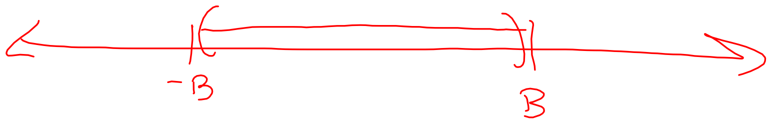
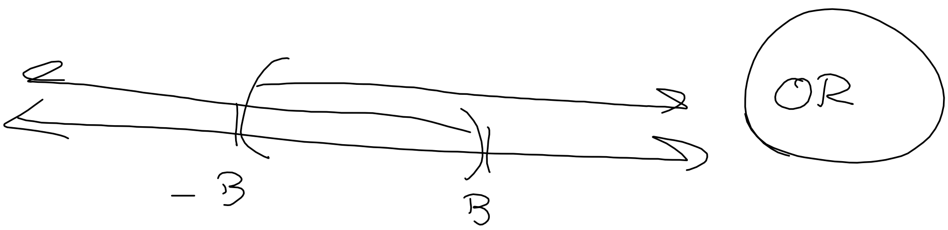


Versus "OR" $A < B$ OR $A > -B$



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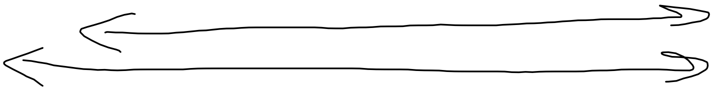


$= (-\infty, \infty)$

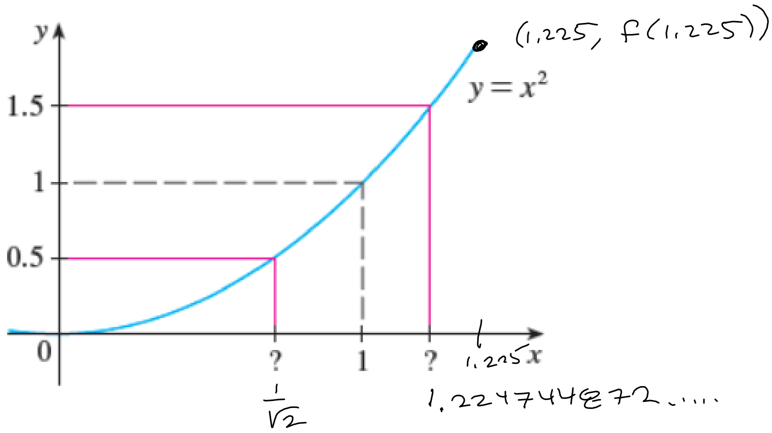
4. 0/1 points SCalc8 1.7.004. [3354157]

Use the given graph of $f(x) = x^2$ to find a number δ such that

if $|x - 1| < \delta$ then $|x^2 - 1| < \frac{1}{2}$.

(Round your answer down to three decimal places.)

$\delta =$ ☐ ✗ 0.224



$x^2 = .5 = \frac{1}{2}$

$x = \pm\sqrt{\frac{1}{2}} = \frac{\pm 1}{\sqrt{2}} = \frac{\pm\sqrt{2}}{2}$ take $+\frac{1}{\sqrt{2}}$

$x^2 = \frac{3}{2} \Rightarrow x = \pm\sqrt{\frac{3}{2}}$ .. .. $\frac{\sqrt{3}}{\sqrt{2}} = \frac{\sqrt{6}}{2}$

$\delta = \min\left\{ \left|1 - \frac{1}{\sqrt{2}}\right|, \left|1 - \sqrt{\frac{3}{2}}\right| \right\}$

$\approx \min\{ |.2928932190|, |-.224744872| \}$

$\approx .225$, but the WebAssign wants to round <u>Down</u>!? why?

Want $|x^2 - 1| < .5$ (Letting $x \to 1$)

$|x-1||x+1| < .5$

$\underbrace{|x-1|}_{<\delta}\ \underbrace{|x+1|}_{\text{How bad can } |x+1| \text{ be?}}$

Assume $\delta \le 1$. Then $|x+1| \le |2+1| = 3$

Define $\delta = \min\{1,$

$|x-1||x+1| < \delta|x+1| < 3\delta \le \varepsilon = .5$

$\delta \le \frac{.5}{3} \approx .1666666667$

6. 0/2 points SCalc8 1.7.007. [3394361]

A graphing calculator is recommended.

For the limit

$f(x) = x^3 - 2x + 6$

$$\lim_{x \to 2} (x^3 - 2x + 6) = 10$$

illustrate the definition by finding the largest possible values of δ that correspond to $\varepsilon = 0.2$ and $\varepsilon = 0.1$. (Round your answers to four decimal places.)

$\varepsilon = 0.2$ $\delta =$ 0.0198 → 0.0198 Makes $f(x) > 10.2$

$\varepsilon = 0.1$ $\delta =$ 0.0099

use Me!

$f(x) = 10.2 \Rightarrow x \approx 2.019764839 \Rightarrow \boxed{\delta \approx .0197}$

$f(x) = 9.8 \Rightarrow x \approx 1.979754912 \Rightarrow \delta \approx .0202$

7. 0/9 points SCalc8 1.7.011. [3354240]

A machinist is required to manufacture a circular metal disk with area 2300 cm^2.

(a) What radius produces such a disk? (Round your answer to four decimal places.)

[ ] ✗ 27.0576 cm   27.05758189

(b) If the machinist is allowed an error tolerance of ±10 cm^2 in the area of the disk, how close to the ideal radius in part (a) must the machinist control the radius? (Round your answers to four decimal places.)

26.9987 ✗ 26.9987 cm < r < 27.1163 ✗ 27.1163 cm

26.99869699   27.11633893

(c) In terms of the ε, δ definition of $\lim_{x \to a} f(x) = L$, what is x?

- area
- target radius
- radius ✓
- target area
- tolerance in the area

✗

What is $f(x)$?

- area ✓
- target radius
- radius
- target area
- tolerance in the area

✗

What is a? (or "limiting value of x!")

- area
- target radius ✓
- radius
- target area
- tolerance in the area

✗

What is L?

- area
- target radius
- radius
- target area ✓
- tolerance in the area

(a) $\pi r^2 = 2300 \equiv L$

$r^2 = \frac{2300}{\pi}$

$r = \pm\sqrt{\frac{2300}{\pi}}$

What value of ε is given?

[ ] ✗ 10 cm^2

What is the corresponding value of δ? (Round your answer to four decimal places.)

[ ] ✗ 0.0587 cm   .05875704

(b) $\varepsilon = 10\ \text{cm}^2$:

$\pi r^2 = 2310$ $\pi r^2 = 2290$

$r^2 = \frac{2310}{\pi}$ $r^2 = \frac{2290}{\pi}$

$r = \pm\sqrt{\frac{2310}{\pi}}$ (Take r>0) $r = \pm\sqrt{\frac{2290}{\pi}}$ (Take r > 0)

≈ 27.11633893   ≈ 26.99869699

round down to be closer to   round up to be closer to

27.05758189

6. 0/2 points

A graphing calculator is recommended.

For the limit

$$\lim_{x \to 2} (x^3 - 2x + 6) = 10$$

Want: $|x^3 - 2x + 6 - 10| < \varepsilon \Rightarrow \underbrace{|x^3 - 2x - 4|}_{\text{Has a factor of } x-2 \text{ in it}} < \varepsilon$

Because we're saying $f(2) = 0$, where

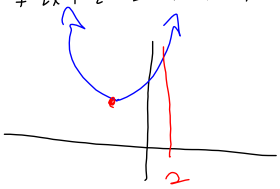
$f(x) = x^3 - 2x - 4$!

$$\begin{array}{r|rrrr} 2 & 1 & 0 & -2 & -4 \\ & & 2 & 4 & 4 \\ \hline & 1x^2 & 2x^1 & 2^c & 0^r \end{array} \qquad (x^3 - 2x - 4) \div (x-2)$$

We want $\underbrace{|x-2|}_{<\delta} \underbrace{|x^2 + 2x + 2|}_{\text{Need a bound on this.}} < \varepsilon$

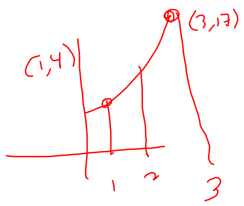
Assume $\delta \le 1$ (Assume we're close to $x = 2$!)

$x^2 + 2x + 2 = x^2 + 2x + 1 - 1 + 2 = (x+1)^2 + 1$



Looks like $x^2 + 2x + 2$ is increasing for $x > -1$

$\delta \le 1 \Rightarrow 1 \le x \le 3$



$1^2 + 1 + 2 = 4 \le x^2 + 2x + 2 \le 3^2 + 2(3) + 2 = 17$

We want $\underbrace{|x-2|}_{<\delta} \underbrace{|x^2 + 2x + 2|}_{<17} < \varepsilon$

$17\delta < \varepsilon$

$\delta < \frac{\varepsilon}{17}$

Proof

Let $\varepsilon > 0$ be given. Define $\delta = \min\{1, \frac{\varepsilon}{17}\}$. Then if

$0 < |x-2| < \delta \Rightarrow |x^3 - 2x + 6 - 10| = |x-2||x^2 + 2x + 2|$

$< \delta |x^2 + 2x + 2| < \delta \cdot 17 \le \frac{\varepsilon}{17} \cdot 17 = \varepsilon$ ▨

This proves $\lim_{x \to 2} (x^3 - 2x + 6) = 10$!