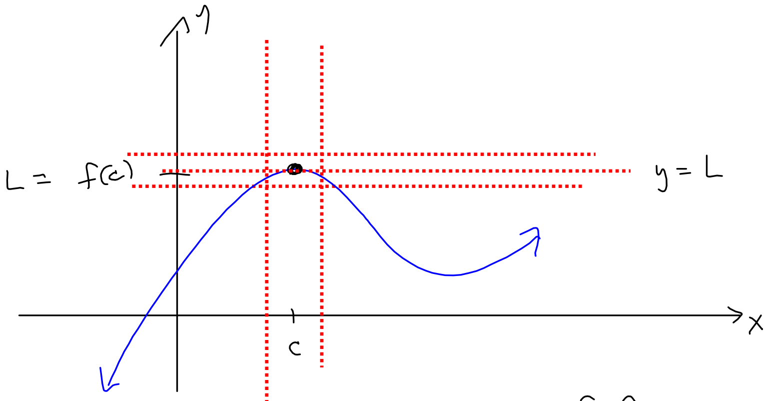


limit as x approaches c of $f(x)$ is L.

$$\lim_{x\to c} f(x) = L.$$

We can make $f(x)$ as close to $y=L$ as we wish, by taking values $\boxed{\text{sufficiently close to } x=c.}$

without touching $x=c$

That's 1.5

§1.6 says "Not only $\lim_{x\to c} f(x)=L$, but $L=f(c)$!"

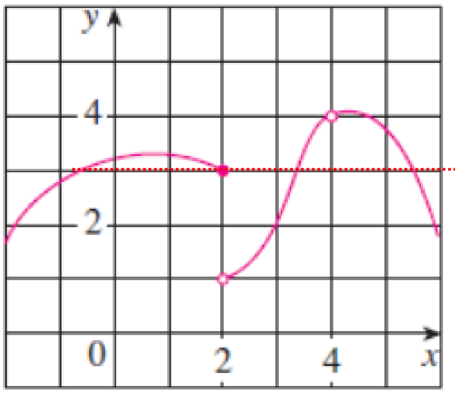
$\lim_{x\to c} f(x) = f(c)$ means $f(x)$ is continuous at $x=c$.

No holes, jumps or $\underbrace{\text{zooming off to } \infty}_{\text{vertical asymptotes}}$

First thing: Go into chat and say "Hi!"

That'll be how we take roll, from now on.

Today: 1.5 and 1.6 is where we're at. Calculus now begins in earnest, with an intuitive discussion of limits and a semi-formal discussion of limit laws (1.5) and continuity (1.6)



$\lim_{x\to 2} f(x)$ DNE $\nexists$

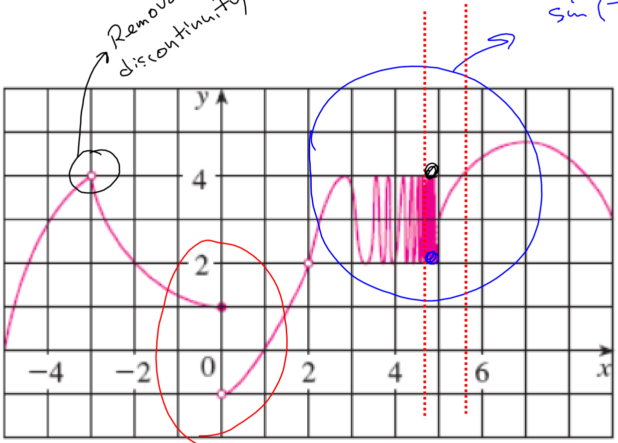
From the left:

$\lim_{x\to 2^-} f(x) = 3$

$\lim_{x\to 2^+} f(x) = 1$

$\lim_{x\to 4} f(x) = 4$ even though there's a hole @ $x=4, y=4$.

Removable discontinuity



$\sin\left(\frac{\pi}{x-5}\right)$ Topologist's sine curve.

$\lim_{x\to 5^-} f(x)$ $\nexists$

$\sin\left(\frac{\pi}{x}\right)$

$g = \begin{cases} x\sin\left(\frac{\pi}{x}\right) & x \neq 0 \\ 0 & \text{at } x = 0 \end{cases}$ is continuous!

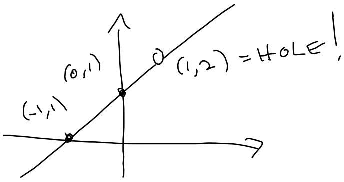
Jump Discontinuity

$\lim_{x\to -3} f(x) = 4$

why do we like the removable discontinuities?

Note: $f(x) = \frac{x^2-1}{x-1}$ $\quad \mathcal{D} = \mathbb{R}\setminus\{1\}$

$f(x) = \frac{x^2-1}{x-1} = \frac{(x+1)\cancel{(x-1)}}{\cancel{x-1}} = x+1$ $\quad (x \neq 1)$

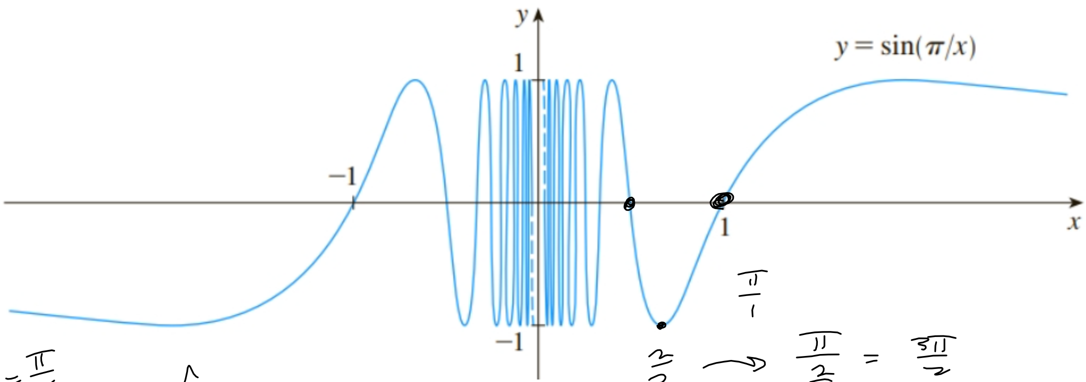


This whole semester is about removing the discontinuity in the difference quotient

$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

$\lim_{x\to c} \frac{f(x)-f(c)}{x-c}$

EXAMPLE 4 Investigate $\lim_{x \to 0} \sin \frac{\pi}{x}$.

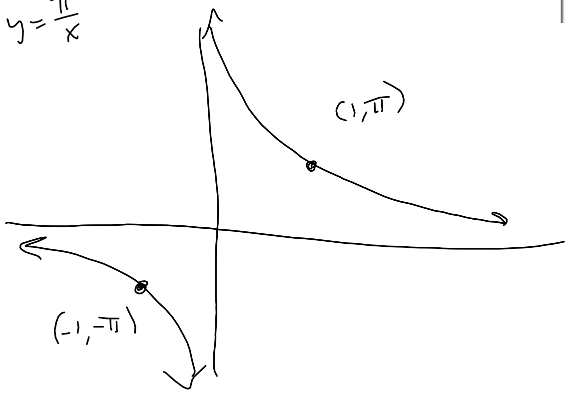


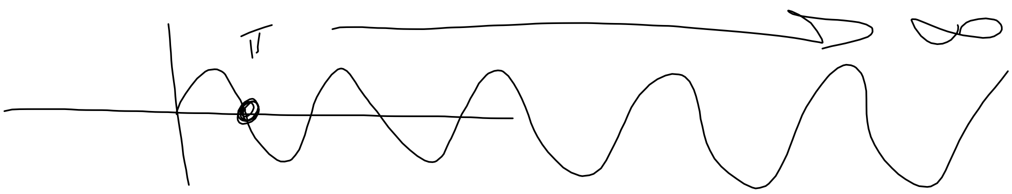
$\frac{\pi}{1}$

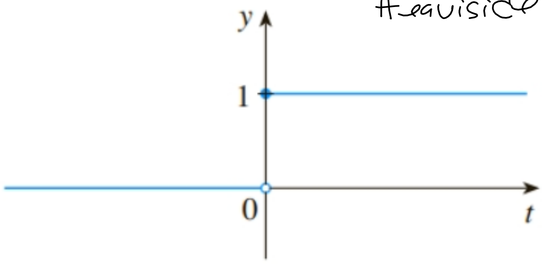
$\frac{2}{3} \rightarrow \frac{\pi}{\frac{2}{3}} = \frac{3\pi}{2}$

$\sin\left(\frac{3\pi}{2}\right) = -1$

$y = \frac{\pi}{x}$







Heaviside Function

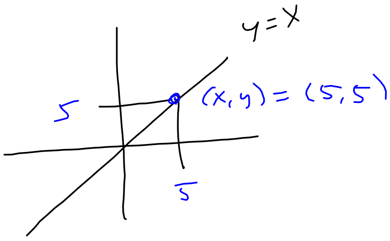
(Check out Dirac Delta Func.)

Dirac δ- function

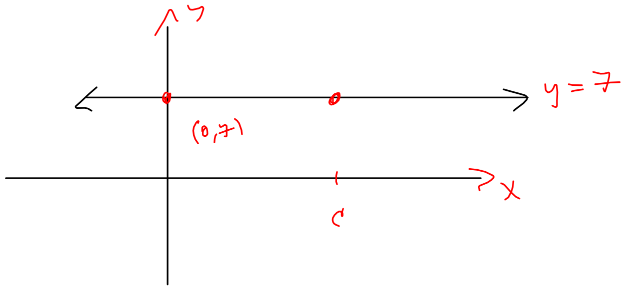
FIGURE 8
The Heaviside function

Basic Limit Ideas :

$$\lim_{x\to c} 7 = 7$$



$$\lim_{x\to 5} x = 5$$



Limits respect addition and multiplication.

$\lim_{x\to c} f(x) = L = \lim f$, $\lim_{x\to c} g(x) = M = \lim g$,

$b, d \in \mathbb{R}$ (i.e. constants)

$$\lim (af) = a \lim f = aL$$

First Glimpse of "Linear Operator."

$$\boxed{\underbrace{\lim (af + bg)}_{\text{The Limit is a LINEAR operation.}} = a \lim f + b \lim g = aL + bM}$$

The Limit is a LINEAR operation.

Since $\lim (fg) = (\lim f)(\lim g) = LM$

$$\lim (f^2) = (\lim f)^2$$

$$\lim (\sqrt{f}) = \sqrt{\lim f}$$

$$\lim_{x\to 2} (5x^2 - 7\sqrt{x} + 2) = 5(2)^2 - 7\sqrt{2} + 2$$

$$= 20 - 7\sqrt{2} + 2 = \boxed{22 - 7\sqrt{2}}$$

$\lim \left(\frac{f}{g}\right) = \frac{\lim f}{\lim g}$, provided $\lim g \neq 0$

A little muddiness.

$\lim_{x\to 1} \frac{1}{x-1} \nexists$

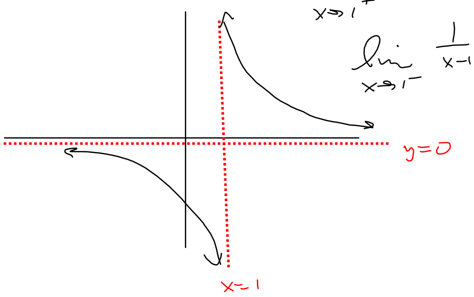
$\lim_{x\to 1^-} \frac{1}{x-1} \nexists$

$\lim_{x\to 1^+} \frac{1}{x-1} \nexists$

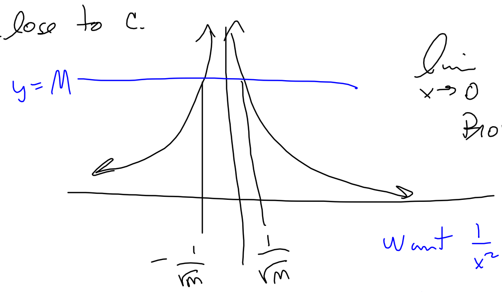
But we also say

$\lim_{x\to 1^+} \frac{1}{x-1} = +\infty$

$\lim_{x\to 1^-} \frac{1}{x-1} = -\infty$



To say $\lim_{x\to c} f(x) = \infty$, means that given ANY Real # M, we can make $f(x) > M$ by taking x sufficiently close to c.



$\lim_{x\to 0} \frac{1}{x^2} = \infty$

Prove it!

Let M be given

want $\frac{1}{x^2} > M$

that means $1 > Mx^2$

$Mx^2 < 1$

$x^2 < \frac{1}{M}$

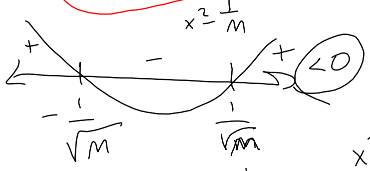
$x^2 - \frac{1}{M} < 0$

Scratch

$x^2 = \frac{1}{M}$

$x = \pm\sqrt{\frac{1}{M}}$

$= \pm \frac{1}{\sqrt{M}}$



$x^2 - \frac{1}{M}$

$-\frac{1}{\sqrt{M}} < x < \frac{1}{\sqrt{M}}$

$x^2 < \frac{1}{M} \Rightarrow$

$-\frac{1}{\sqrt{M}} < x < \frac{1}{\sqrt{M}}$

As long as we're closer to $x=0$ than $\frac{1}{\sqrt{M}}$, we'll always be above $y = M$.

We're testing on WebAssign, this semester. I've done a ton of work on Test-Prep videos,

but we're not testing that way, any more. About all their good for is to give you a summary of the chapter in terms of the kinds of questions I would normally ask on a traditional test.