Section 1.1 #21 (on WebAssign)

21. 0/1 points

Evaluate the difference quotient for the given function. Simplify your answer.

$$f(x)=\frac{x+7}{x+2},\quad \frac{f(x)-f(3)}{x-3}$$

Look for the Poll.

$$f(x)=\frac{x+7}{x+2},\quad \frac{f(x)-f(3)}{x-3}$$

In general, $\frac{f(x)-f(c)}{x-c}=\frac{\frac{x+7}{x+2}-\frac{c+7}{c+2}}{x-c}=$

$$\frac{\frac{(x-7)(c+7)-(x+2)(c+7)}{(x+2)(c+2)}}{\frac{x-c}{1}}=$$

$$\frac{xc+7x-7c-49-(xc+7x+2c+14)}{(x+2)(c+2)}\cdot\frac{1}{x-c}$$

$$=\frac{cx+7x-7c-49-cx-7x-2c-14}{(x-c)(x+2)(c+2)}=\frac{-9c-63}{(x-c)(x+2)(c+2)}\qquad -9c-63=-9(c+7)$$

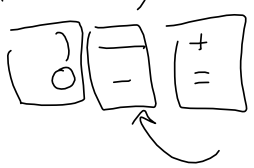
Not seeing it. Should be able to cancel the x-c!

$$f(x)=\frac{x+7}{x+2}\Rightarrow f(3)=\frac{3+7}{3+2}=\frac{10}{5}=2$$

$$\frac{f(x)-f(3)}{x-3}=\frac{\frac{x+7}{x+2}-2}{x-3}=\frac{\frac{x+7-2(x+2)}{x+2}}{x-3}$$

$$=\frac{x+7-2x-4}{(x+2)(x-3)}=\frac{-x+3}{(x+2)(x-3)}=\frac{-\cancel{(x-3)}}{(x+2)\cancel{(x-3)}}=\frac{-1}{x+2}$$

To enter a "−",



$f(x) = \frac{x+7}{x+2}$, $\frac{f(x) - f(3)}{x-3}$

Sketch it !

$\mathcal{D}$: $x+2 \neq 0$ $\quad \mathcal{D}(f) = \{x \mid x \neq -2\}$

$x \neq -2$ $\quad \boxed{V.A.:\ x = -2}$

$f(x) = 0 \Rightarrow x + 7 = 0$

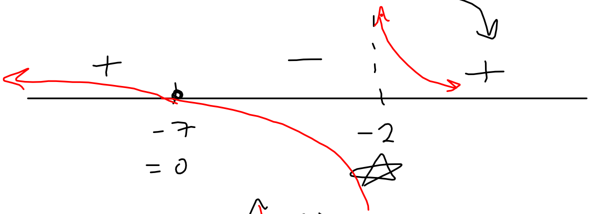
$x = -7 \rightsquigarrow \boxed{(-7, 0)\ \text{x-int}}$

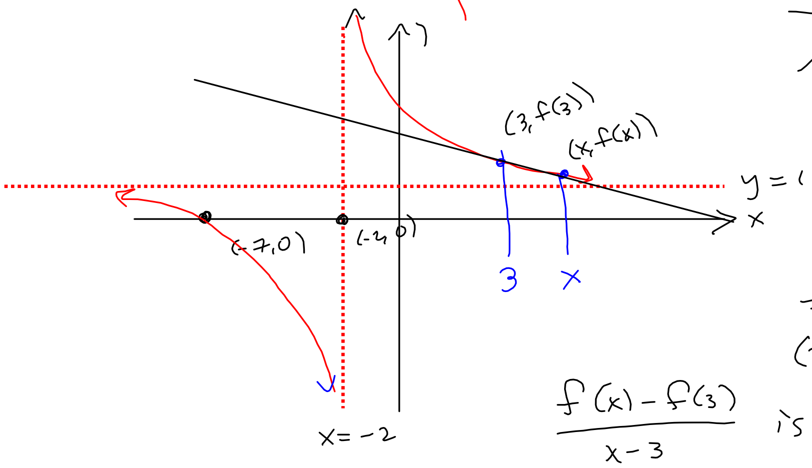
y-int : $f(0) = \frac{7}{2} \rightsquigarrow \boxed{(0, \frac{7}{2})\ \text{y-int}}$

H.A. = Horizontal Asymptote $x \to \pm\infty$ $(|x| \to \infty)$

$\frac{x+7}{x+2} \xrightarrow{x \to \infty} \frac{x}{x} = \boxed{1 = y \quad H.A.}$

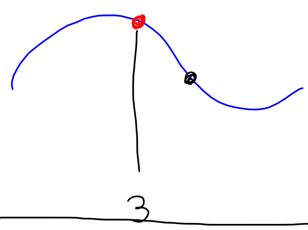
$\frac{x+7}{x+2}$





Secant line through $(3, f(3))$ and $(x, f(x))$

$\frac{f(x) - f(3)}{x-3}$ is its slope.



The whole point of differential calculus is to take x arbitrarily close to 3.

"Pass to the limit as x approaches 3."

$\boxed{\lim_{x \to c} f(x) = f(c)}$ continuous.

$\boxed{\lim_{x \to 3} \frac{f(x) - f(3)}{x-3}}$ = slope of $f(x)$ @ $x = 3$.

Differentiable means THIS limit exists.

Section 1.4 - Approaching the tangent line by gentle intro to notion of "limit."

8. The displacement (in centimeters) of a particle moving back and forth along a straight line is given by the equation of motion $s = 2\sin(\pi t) + 3\cos(\pi t)$, where t is measured in seconds.

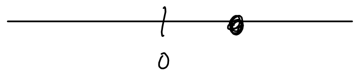
(a) Find the average velocity during each time period:

(i) [1, 2] (ii) [1, 1.1]

(iii) [1, 1.01] (iv) [1, 1.001]

(b) Estimate the instantaneous velocity of the particle when $t = 1$.

We get closer and closer to $m_{avg} = -2\pi$



See Maple for the implementation on x^2.

See Spreadsheet for implementation, also.

Equation of tangent line to a curve at a point.

$y = m(x - x_1) + y_1$

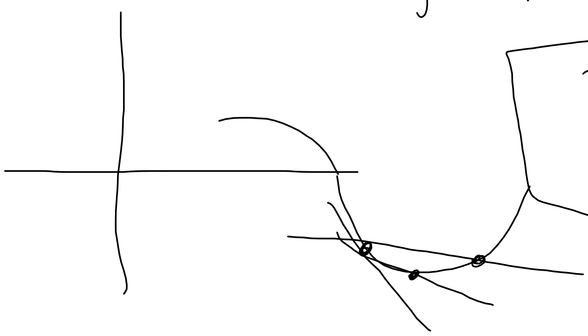
$y = m(x - x_1) + f(x_1)$

$y = f'(x_1)(x - x_1) + f(x_1)$

$(x_1, y_1) = (1, -3)$

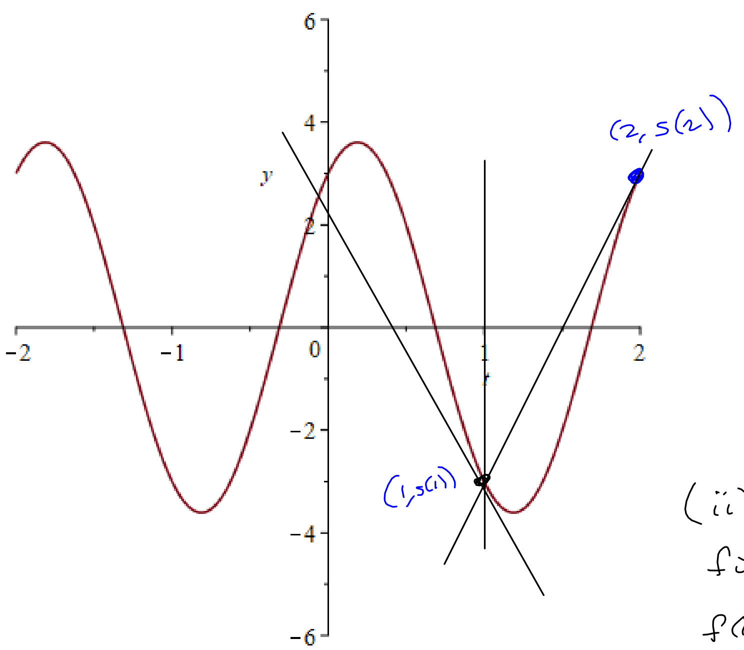
$y = -2\pi(x-1) + (-3)$

$= -2\pi(x-1) - 3$



Tangent Slope is

$$m_{tan} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$



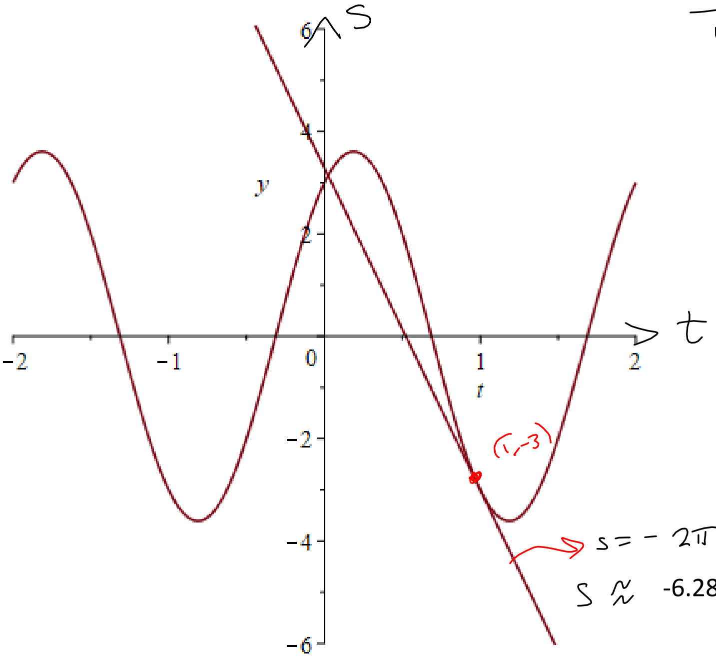
(i) $\dfrac{s(2)-s(1)}{2-1}$

$2 = 1+1$, so $h = 1$

$$\frac{s(2)-s(1)}{2-1} = \frac{s(1+1)-s(1)}{1} = \frac{s(1+h)-s(1)}{h}$$

(ii) $[1, 1.1]$ means find m_{avg} on the interval

$$\frac{f(1.1)-f(1)}{1.1-1} = \frac{f(1+.1)-f(1)}{.1}$$



$s = -2\pi(t-1) - 3$

$S \approx$ -6.283187304 $(t-1) - 3)$

Do the same for x^2:

Algebraic Solution

$$f(x) = x^2 \Rightarrow \frac{f(x+h)-f(x)}{h} = \frac{(x+h)^2 - x^2}{h}$$

$$= \frac{x^2+2xh+h^2-x^2}{h} = \frac{2xh+h^2}{h} = \frac{h(2x+h)}{h}$$

$$= 2x+h \xrightarrow{h\to 0} 2x = \text{slope of } x^2 \text{ @ } x.$$

$f'(x) = 2x$ is the "DERIVATIVE!"

what's the slope of $f(x) = x^2$ @ $x=1$?

$$f'(1) = 2(1) = 2.$$

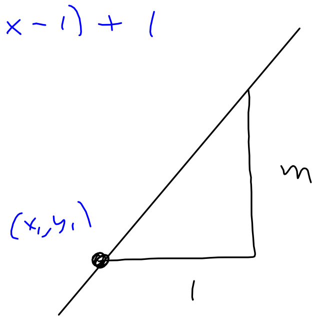
TANGENT LINE (Linearization)

$$L(x) = f'(1)(x-1) + f(1)$$

$$= m(x-x_1) + f(x_1)$$

$$= 2(x-1) + 1$$

$$f(1) = 1^2 = 1$$



33–38 Find the functions (a) $f \circ g$, (b) $g \circ f$, (c) $f \circ f$, and (d) $g \circ g$ and their domains.

33. $f(x) = 3x + 5, \quad g(x) = x^2 + x$

34. $f(x) = x^3 - 2, \quad g(x) = 1 - 4x$

35. $f(x) = \sqrt{x + 1}, \quad g(x) = 4x - 3$

36. $f(x) = \sin x, \quad g(x) = x^2 + 1$

37. $f(x) = x + \dfrac{1}{x}, \quad g(x) = \dfrac{x+1}{x+2}$

38. $f(x) = \dfrac{x}{1+x}, \quad g(x) = \sin 2x$

#38 $f \circ g$:

$$f(g(x)) = \frac{g(x)}{1+g(x)} = \frac{\sin(2x)}{1+\sin(2x)}$$

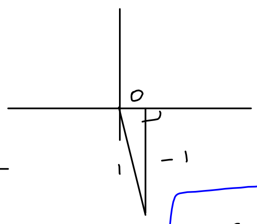
$\mathcal{D}(f \circ g)$: Need: $\sin(2x) + 1 \neq 0$

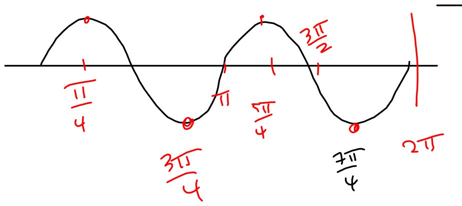
$\sin(2x) + 1 = 0$

$\sin(2x) = -1 \implies$

$2x = \frac{3\pi}{2}$

$x = \frac{3\pi}{4}$





$\implies$ $\boxed{x \in \left\{ \frac{3\pi}{4} + n\pi \;\middle|\; n \in \mathbb{Z} \right\} = A}$

$\boxed{\mathcal{D}(f \circ g) = \mathbb{R} \setminus A}$

$\mathcal{D}(f \circ g) = \mathcal{D}(f(g(x)))$

$= \{ x \mid x \in \mathcal{D}(g) \text{ and } g(x) \in \mathcal{D}(f) \}$

§1.3

49–51 Express the function in the form $f \circ g \circ h$.

49. $R(x) = \sqrt{\sqrt{x} - 1}$ **50.** $H(x) = \sqrt[8]{2 + |x|}$

51. $S(t) = \sin^2(\cos t)$

51 $\left(\sin(\cos(t))\right)^2$

$f(x) = x^2$

$g(w) = \sin(w)$

$h(t) = \cos(t)$

Then $\sin^2(\cos(t)) = f \circ g \circ h$

$\sqrt{\sqrt{x} - 1}$

$f(x) = \sqrt{x}$

$g(x) = x - 1$

$h(x) = \sqrt{x}$

$f \circ g \circ h = f(g(h(x))) = f(g(\sqrt{x})) = f(\sqrt{x} - 1)$

$= \sqrt{\sqrt{x} - 1}$

$f(x) = \sqrt{\sqrt{x} - 1}$

$g(x) = x$, $h(x) = x$

Important for Chain Rule, Later.

§1.4 is to convince you that you
don't want to work things like §1.4 works them

Next up: §1.5, 1.6 & Questions

I want you to "run" the lecture by
asking about stuff, rather than waiting for
me to magically say what you need to hear.